

# Hamiltonian Unification of General Relativity and Standard Model

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## Abstract

The Hamiltonian approach to the General Relativity and the Standard Model is studied in the context of its consistency with the Newton law, the Higgs effect, the Hubble cosmological evolution and the Cosmic Microwave Background radiation physics.

The version of the Higgs potential is proposed, where its constant parameter is replaced by the dynamic zeroth Fourier harmonic of the very Higgs field. In this model, the extremum of the Coleman–Weinberg effective potential obtained from the unit vacuum–vacuum transition amplitude immediately predicts mass of Higgs field and removes tremendous vacuum cosmological density.

We show that the relativity principles unambiguously treat the Planck epoch, in the General Relativity, as the present-day one. It was shown that there are initial data of the Electro-Weak epoch compatible with supposition that all particles in the Universe are final products of decays of primordial Higgs particles and W-, Z-vector bosons created from vacuum at the instant treated as the "Big-Bang".

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## Introduction

The unification problem of the General Relativity (GR) and the Standard Model (SM) is one of main questions of modern physics. The main difficulty of this unification lies in the different theoretical levels of their presentation: quantum for SM and classical for GR. However, both these theories have common roots of their origin (mechanics and electrodynamics) and *principles of relativity*, and both they are in agreement with observational and experimental data.

It is worthwhile to recall here these common *principles of relativity* and their relation to observational and experimental data. Actually, physics arises as a science about measurements and observations. It supposes two distinguished reference frames - the observer *rest frame* and the observable *comoving frame*. In particular, in modern cosmology, comoving frame of the Universe is identified with the Cosmic Microwave Background radiation frame that differs from the rest frame by the nonzero dipole component of the temperature fluctuations [1].

*Differences* between these two frames underlie all principles of relativity including the Copernicus – Galilei relativity as a *difference* of initial positions and velocities, and the Poincaré – Einstein special relativity (SR) [2, 3] as a *difference* of measurable times in different frames. Principles of relativity mean that there are *degrees of freedom* together with their motion equations and initial data that are free from these equations. The Copernicus – Galilei relativity means that these *degrees of freedom* are spatial coordinates of a particle. Equations of motion as invariants of the Galilei group and the manifold of initial data are the main concepts of the *first physical theory* created by Isaac Newton.

The Poincaré – Einstein Special Relativity (SR) means that the time coordinate is the *degree of freedom* of a particle too, so that the complete set of *degrees of freedom* forms the Minkowski *space of events*. A geometric interval on the line of a particle in this space of events is formed by its metric with a single component (the *lapse function*), and there are *reparametrizations of a coordinate evolution parameter*. The Hilbert action-interval variational principle provides the *lapse function* equation as the *energy constraint*. A solution of this *constraint* with respect to the time-like variable momentum gives *energy in space of events* and the relation of this time-like variable with *geometric interval*. The primary and secondary quantization of the *energy constraint* give Quantum Field Theory (QFT) where the vacuum as the state with minimal *energy in space of events* is postulated with definite traffic rules of the motion of a particle in its *space of events*. The complete set of these results can be described by the geometro-dynamic action principle formulated by David Hilbert in his *Foundations of Physics* [4] in the Einstein *General Relativity* (GR).

Recall that the Hilbert geometro-dynamics includes a geometric interval as an additional reference quantity, and *general coordinate transformations* are considered as diffeomorphisms of coordinates and variables [4, 5] similar to *reparametrizations of a coordinate evolution parameter* in SR [6, 7, 8].

Actually, Hilbert's "Foundations of Physics" for General Relativity and Quantum Field Theory give hopes for an opportunity to construct a realistic quantum theory for GR. These hopes are based, on the one hand, on the existence of the Hilbert-type geometric formulation [4] of SR with the energy constraint considered as the simplest model of GR and, on the other hand, on the contemporary QFT based on the primary and secondary quantization of this energy constraint [8, 9, 10].

One can reconstruct a direct pathway from geometry of a relativistic particle in SR to the causal operator quantization of fields of these particles and their quantum creation from a vacuum in order to formulate a similar direct way from geometry of GR [4] to the causal operator quantization of universes and to their quantum creation from a vacuum treated as a state with the minimal *energy of events*. This formulation includes

1. the Wheeler–DeWitt definition of the *field space of events* [11], where diffeomorphisms are split from transformations of the frames of references using the Fock *simplex of reference* [12];
2. the choice of the Dirac specific *reference frame* [13];
3. resolving the energy constraint in the class of functions of the gauge transformations established by Zel'manov [14];
4. a treatment of the cosmological scale factor as a zeroth mode field variable [6, 7];
5. the constraint-shell values of the action and geometric interval [15] in terms of diffeo-invariant variables;
6. and the notions of *energy, time, particle and universe, number of particles and number of universes* defined by the low-energy expansion of this *reduced* action following the correspondence principle with nonrelativistic theory in SR [2, 3].

Thus, further theoretical developments of GR and QFT are convergent, in spite of the accepted opinion that quantum formulation of GR can not exist. At the present time, there is a set of theoretical and observational arguments in favor of the opposite opinion: GR [4, 5] has a consistent interpretation only in the form of quantum theory of the type of the microscopic theory of superfluidity [16, 17, 18] with the Bogoliubov transformations used for construction of integrals of motion and stable physical states including a vacuum [9, 10].

In any case, the Hamiltonian pathway of SR towards QFT can be repeated for GR, because the dimension of the diffeomorphism group of Dirac–Arnowitt–Deser–Misner Hamiltonian approach to GR coincides with the dimension of constraints removing a part of canonical momenta in accordance with the second Nöther theorem [9, 10].

Thus, *the Hamiltonian approach to both the GR and SM can be the basis of their unification.*

This Hamiltonian unification of GR and SM is just the topic of the present paper. We show that the final GR&SM theory is a *conformal relativistic brane* in  $D = 4$  space-time (internal coordinates) moving in field space of events forming by dilaton and scalar, spinor, and vector fields of SM. The zeroth mode sector of the GR&SM theory forms a cosmological model with free initial data at the beginning of the Universe. There are initial data that describe the vacuum creation of SM particles in agreement with Supernovae (SN) data, CMB physics, and the present day energy budget of the Universe.

We show that the Hamiltonian presentation of the GR&SM brane differs from the acceptable approach to a relativistic brane [19, 20, 21]. The first difference is that the zeroth modes (zeroth Fourier harmonics) are completely separated from the nonzero ones and their interference term disappears in the energy constraint, so that the constraint algebra differs from the Virasoro one (in the string theory the Hamiltonian method corresponds to the R  hrlich gauge [22, 23]). In the opposite case of the Virasoro algebra, we have the double counting of the zeroth mode destructing the Hamiltonian presentation of the GR&SM brane. The second difference are the diffeo-invariant observables including the conformal time, in contrast to the naive diffeo-variant formulation of GR [24, 25], where the coordinate time as the object of reparametrizations is confused with the reparametrization-invariant conformal time treated as an observable quantity in cosmology. The third difference is the Weyl principle of relativity of units [26]. In accord with this principle we can measure only a ratio of a measurable interval and units of measurement of the interval.

Thus, the Hamiltonian formulation of the GR&SM theory keeps all concepts that were worked out in modern relativistic and quantum physics, including the first and second N  ther theorems [27], *space of events, energy, time of events, time-interval, vacuum postulate, Wigner representation of the Poincar   group, Hamiltonian reduction.*

All these relativity principles mean that the cosmological scale factor can be a "degree of freedom" with free initial data fitted by observations [8, 10, 15, 28, 29]. Recall that, in the Inflationary Model [24], the initial data of the cosmological scale factor is identified with the Planck scale.

The topic of the present paper is the Hamiltonian GR&SM unification, where the Universe is identified with classical and quantum solutions of equations of motion with "free diffeo-invariant initial data"

in the CMB reference frame.

Section 1 is devoted to the Hamiltonian approach to SM. Section 2 is devoted to the Hamiltonian approach to GR. In Section 3, we consider the problem of unification of GR and SM compatible with the Newton law in GR and spontaneous symmetry breaking in SM. The identification of GR and SM with a brane is considered in Section 4. Section 5 is devoted to observational tests of unified theory.



# 1 Hamiltonian approach to gauge theories

The Hamiltonian approach to gauge theories was considered as the mainstream of the development of gauge theories beginning with the pioneer papers by Paul Dirac [30], Werner Heisenberg, Wolfgang Pauli [31], and finishing by Julian Schwinger's quantization of the non-Abelian theory [32] (see in detail [33, 34, 35, 36]). They postulated the higher priority of the quantum principles, in particular, in accordance with the uncertainty principle, one counted that we cannot quantize "field variables" whose velocities are absent in the Lagrangian. Therefore, vector field time components with negative contributions to energy are eliminated, as it was accepted in the Dirac approach to QED [30]. This illumination leads to static interactions and instantaneous bound states.

Remember that the Dirac Hamiltonian approach generalized to the non-Abelian theory [32, 36] and the massive vector fields [35] provide the fundamental operator quantization and correct relativistic transformations of states of quantized fields. This Hamiltonian approach is considered [34] as the foundation of all heuristic methods of quantization of gauge theories, including the Faddeev-Popov (FP) method [37] used now for description of the Standard Model of elementary particles [38]. Moreover, Schwinger ... *rejected all Lorentz gauge formulations as unsuited to the role of providing the fundamental operator quantization* (see [32] p.324). However, a contemporary reader could not find the Hamiltonian presentation of the Standard Model (SM) because there is an opinion [34] that this presentation is completely equivalent to the accepted version of SM based on the FP method [38].

## 1.1 Quantum Electrodynamics

### 1.1.1 Action and reference frame

Let us recall the Dirac approach [30] to QED. The theory is given by the well known action

$$S = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} [i\partial - m] \psi + A_\mu j^\mu \right\}, \quad (1.1)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is tension,  $A_\mu$  is a vector potential,  $\psi$  is the Dirac electron-positron bispinor field, and  $j_\mu = e\bar{\psi}\gamma_\mu\psi$  is the charge current,  $\partial = \partial^\mu\gamma_\mu$ . This action is invariant with respect to the collection of gauge transformations

$$A_\mu^\lambda = A_\mu + \partial_\mu \lambda, \quad \psi^\lambda = e^{+ie\lambda} \psi. \quad (1.2)$$

The action principle used for the action (1.1) gives the Euler-Lagrange equations of motion - known as the Maxwell equations

$$\partial_\nu F^{\mu\nu} + j^\mu = 0, \quad (1.3)$$

Physical solutions of the Maxwell equations are obtained in a fixed *inertial reference frame* distinguished by a unit timelike vector  $n_\mu$ . This vector splits the gauge field  $A_\mu$  into the timelike  $A_0 = A_\mu n_\mu$  and spacelike  $A_\nu^\perp = A_\nu - n_\nu(A_\mu n_\mu)$  components. Now we rewrite the Maxwell equations in terms of components

$$\Delta A_0 - \partial_0 \partial_k A_k = j_0, \quad (1.4)$$

$$\square A_k - \partial_k [\partial_0 A_0 - \partial_i A_i] = -j_k. \quad (1.5)$$

The field component  $A_0$  cannot be a *degree of freedom* because its canonical conjugate momentum vanishes. The Gauss constraints (1.4) have the solution

$$A_0 + \partial_0 \Lambda = -\frac{1}{4\pi} \int d^3 y \frac{j_0(x_0, y_k)}{|\mathbf{x} - \mathbf{y}|}, \quad (1.6)$$

where

$$\Lambda = -\frac{1}{\Delta} \partial_k A_k = \frac{1}{4\pi} \int d^3 y \frac{\partial_k A_k}{|\mathbf{x} - \mathbf{y}|} \quad (1.7)$$

is a longitudinal component. The result (1.6) is treated as the *Coulomb potential field* leading to the *static* interaction.

### 1.1.2 Elimination of time component

Dirac [30] proposed to eliminate the time component by substituting the manifest resolution of the Gauss constraints given by (1.6) into the initial action (1.1). This substitution - known as the reduction procedure - allows us to eliminate nonphysical pure gauge degrees of freedom [72]. After this step the action (1.1) takes the form

$$S = \int d^4 x \left\{ \frac{1}{2} (\partial_\mu A_k^\text{T})^2 + \bar{\psi} [i\partial - m] \psi - j_0 \partial_0 \Lambda - A_k^\text{T} j_k + \frac{1}{2} j_0 \frac{1}{\Delta} j_0 \right\}, \quad (1.8)$$

where

$$A_k^\text{T} = \left( \delta_{ij} - \frac{\partial_i \partial_j}{\Delta} \right) A_j. \quad (1.9)$$

This substitution leaves the longitudinal component  $\Lambda$  given by Eq. (1.7) without any kinetic term.

There are two possibilities. The first one is to treat  $\Lambda$  as the Lagrange factor that leads to the conservation law (1.3). In this approach, the longitudinal component is treated as an independent variable. This treatment violates gauge invariance because this component is gauge-variant and it cannot be measurable. Moreover, the time derivative of the longitudinal component in Eq. (1.6) looks like a physical source of the Coulomb potential. By these reasons we will not consider this approach in this paper.

In the second possibility, a measurable potential stress is identified with the gauge-invariant quantity (1.6)

$$A_0^R = A_0 - \frac{\partial_0 \partial_k}{\Delta} A_k, \quad (1.10)$$

This approach is consistent with the principle of gauge invariance that identifies observables with gauge-invariant quantities. Therefore, according to the gauge-invariance, the longitudinal component should be eliminated from the set of degrees of freedom of QED too.

### 1.1.3 Elimination of longitudinal component

This elimination is fulfilled by the choice of the "radiation variables" as gauge invariant functionals of the initial fields, *i.e.* "dressed fields" [30]

$$A_\mu^R = A_\mu + \partial_\mu \Lambda, \quad \psi^R = e^{ie\Lambda} \psi, \quad (1.11)$$

In this case, the linear term  $\partial_k A_k$  disappears in the Gauss law (1.4)

$$\Delta A_0^R = j_0^R \equiv e \bar{\psi}^R \gamma_0 \psi^R. \quad (1.12)$$

The source of the gauge-invariant *potential field*  $A_0^R$  can be only an electric current  $j_0^R$ , whereas the spatial components of the vector field  $A_k^R$  coincide with the transversal one

$$\partial_k A_k^R = \partial_k A_k^T \equiv 0. \quad (1.13)$$

In this manner the frame-fixing  $A_\mu = (A_0, A_k)$  is compatible with understanding of  $A_0$  as a classical field and the use of the Dirac dressed fields (1.11) of the Gauss constraints (1.4) leads to understanding of the variables (1.11) as gauge-invariant functionals of the initial fields.

### 1.1.4 Static interaction

Substitution of the manifest resolution of the Gauss constraints (1.4) into the initial action (1.1) calculated on constraints leads to that the initial action can be expressed in terms of the gauge-invariant radiation variables (1.11) [30, 33]

$$S = \int d^4x \left\{ \frac{1}{2} (\partial_\mu A_k^R)^2 + \bar{\psi}^R [i \not{\partial} - m] \psi^R - A_k^R j_k^R + \frac{1}{2} j_0^R \frac{1}{\Delta} j_0^R \right\} \quad (1.14)$$

The Hamiltonian, which corresponds to this action, has the form

$$\mathcal{H} = \frac{(\Pi_k^R)^2 + (\partial_j A_k^R)^2}{2} + p_\psi^R \gamma_0 [i \gamma_k \partial_k + m] \psi^R + A_k^R j_k^R - \frac{1}{2} j_0^R \frac{1}{\Delta} j_0^R \quad (1.15)$$

where  $\Pi_k^R, p_\psi^R$  are the canonical conjugate momentum fields of the theory calculated in a standard way. Hence the vacuum can be defined as a state with minimal energy obtained as the value of the Hamiltonian for the equations of motion. Relativistic covariant transformations of the gauge-invariant fields are proved on the level of the fundamental operator quantization in the form of the Poincaré algebra generators [32]. The status of the theorem of equivalence between the Dirac radiation variables and the Lorentz gauge formulation is considered in [36, 35].

### 1.1.5 Comparison of radiation variables with the Lorentz gauge ones

The static interaction and the corresponding bound states are lost in any frame free formulation including the Lorentz gauge one. The action (1.8) transforms into

$$S = \int d^4x \left\{ -\frac{1}{2}(\partial_\mu A_\nu^L)^2 + \bar{\psi}^L[i\partial - m]\psi^L + A_\mu^L j^{L\mu} \right\}, \quad (1.16)$$

where

$$A_\mu^L = A_\mu + \partial_\mu \Lambda^L, \quad \psi^L = e^{ie\Lambda^L} \psi, \quad \Lambda^L = -\frac{1}{\square} \partial^\mu A_\mu^L \quad (1.17)$$

are the manifest gauge-invariant functionals satisfying the equations of motion

$$\square A_\mu^L = -j_\mu^L, \quad (1.18)$$

with the current  $j_\mu^L = -e\bar{\psi}^L \gamma_\mu \psi^L$  and the gauge constraints

$$\partial_\mu A^{L\mu} \equiv 0. \quad (1.19)$$

Really, instead of the potential (satisfying the Gauss constraints  $\triangle A_0^R = j_0^R$ ) and two transverse variables in QED in terms of the radiation variables (1.11) we have here three independent dynamic variables, one of which  $A_0^L$  satisfies the equation

$$\square A_0^L = -j_0, \quad (1.20)$$

and gives a negative contribution to the energy.

We can see that there are two distinctions of the “Lorentz gauge formulation” from the radiation variables. The first is the loss of Coulomb poles (*i.e.* static interactions). The second is the treatment of the time component  $A_0$  as an independent variable with the negative contribution to the energy; therefore, in this case, the vacuum as the state with the minimal energy is absent. In other words, one can say that the static interaction is the consequence of the vacuum postulate too. The inequivalence between the radiation variables and the

Lorentz ones does not mean violation of the gauge invariance, because both the variables can be defined as the gauge-invariant functionals of the initial gauge fields (1.11) and (1.17).

In order to demonstrate the inequivalence between the radiation variables and the Lorentz ones, let us consider the electron-positron scattering amplitude  $T^R = \langle e^+, e^- | \hat{S} | e^+, e^- \rangle$ . One can see that the Feynman rules in the radiation gauge give the amplitude in terms of the current  $j_\nu = \bar{e}\gamma_\nu e$

$$\begin{aligned} T^R &= \frac{j_0^2}{\mathbf{q}^2} + \left( \delta_{ik} - \frac{q_i q_k}{\mathbf{q}^2} \right) \frac{j_i j_k}{q^2 + i\varepsilon} \\ &\equiv \frac{-j^2}{q^2 + i\varepsilon} + \boxed{\frac{(q_0 j_0)^2 - (\mathbf{q} \cdot \mathbf{j})^2}{\mathbf{q}^2 [q^2 + i\varepsilon]}}. \end{aligned} \quad (1.21)$$

This amplitude coincides with the Lorentz gauge one

$$T^L = -\frac{1}{q^2 + i\varepsilon} \left[ j^2 - \boxed{\frac{(q_0 j_0 - \mathbf{q} \cdot \mathbf{j})^2}{q^2 + i\varepsilon}} \right] \quad (1.22)$$

when the box terms in Eq. (1.21) can be eliminated. Thus, the Faddeev equivalence theorem [34] is valid if the currents are conserved

$$q_0 j_0 - \mathbf{q} \cdot \mathbf{j} = qj = 0, \quad (1.23)$$

However, for the action with the external sources the currents are not conserved. Instead of the classical conservation laws we have the Ward–Takahashi identities for Green functions, where the currents are not conserved

$$q_0 j_0 - \mathbf{q} \cdot \mathbf{j} \neq 0. \quad (1.24)$$

In particular, the Lorentz gauge perturbation theory (where the propagator has only the light cone singularity  $q_\mu q^\mu = 0$ ) can not describe instantaneous Coulomb atoms; this perturbation theory contains only the Wick–Cutkosky bound states whose spectrum is not observed in the Nature.

Thus, we can give a response to the question: What are new physical results that follow from the Hamiltonian approach to QED in comparison with the frame-free Lorentz gauge formulation? In the framework of the perturbation theory, the Hamiltonian presentation of QED contains the static Coulomb interaction (1.21) forming instantaneous bound states observed in the Nature, whereas all frame free formulations lose this static interaction together with instantaneous bound states in the lowest order of perturbation theory on retarded interactions called the radiation correction. Nobody has proved that the sum of these retarded radiation corrections with the light-cone singularity propagators (1.22) can restore the Coulomb interaction that was removed from propagators (1.21) by hand on the level of the action.

## 1.2 Vector bosons theory

### 1.2.1 Lagrangian and reference frame

The classical Lagrangian of massive QED is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2V_\mu^2 + \bar{\psi}(i\partial - m)\psi + V_\mu j^\mu, \quad (1.25)$$

In a fixed reference frame this Lagrangian takes the form

$$\begin{aligned} \mathcal{L} = & \frac{(\dot{V}_k - \partial_k V_0)^2 - (\partial_j V_k^T)^2 + M^2(V_0^2 - V_k^2)}{2} + \\ & + \bar{\psi}(i\partial - m)\psi + V_0 j_0 - V_k j_k, \end{aligned} \quad (1.26)$$

where  $\dot{V} = \partial_0 V$  and  $V_k^T$  is the transverse component defined by the action of the projection operator given in Eq. (1.9). In contrast to QED this action is not invariant with respect to gauge transformations. Nevertheless, from the Hamiltonian viewpoint the massive theory has the same problem as QED. The time component of the massive boson has a vanishing canonical momentum.

### 1.2.2 Elimination of time component

In [35] one supposed to eliminate the time component from the set of degrees of freedom like the Dirac approach to QED, *i.e.*, using the action principle. In the massive case it produces the equation of motion

$$(\Delta - M^2)V_0 = \partial_i \dot{V}_i + j_0. \quad (1.27)$$

which is understood as constraints and has the solution

$$V_0 = \left( \frac{1}{\Delta - M^2} \partial_i V_i \right) + \frac{1}{\Delta - M^2} j_0. \quad (1.28)$$

In order to eliminate the time component, let us insert (1.28) into the Lagrangian (1.26) [30, 35]

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \left[ (\dot{V}_k^T)^2 + V_k^T (\Delta - M^2) V_k^T + j_0 \frac{1}{\Delta - M^2} j_0 \right] + \\ & + \bar{\psi}(i\partial - m)\psi - V_k^T j_k \\ & + \frac{1}{2} \left[ \dot{V}_k^{\parallel} M^2 \frac{1}{\Delta - M^2} \dot{V}_k^{\parallel} - M^2 (V_k^{\parallel})^2 \right] - V_k^{\parallel} j_k + \\ & + j_0 \frac{1}{\Delta - M^2} \partial_k \dot{V}_k^{\parallel}, \end{aligned} \quad (1.29)$$

where we decomposed the vector field  $V_k = V_k^T + V_k^{\parallel}$  by means of the projection operator by analogy with (1.9). The last two terms are the contributions of the longitudinal component only. This Lagrangian

contains the longitudinal component which is the dynamical variable described by the bilinear term. Now we propose the following transformation:

$$\begin{aligned}\bar{\psi}(i \not{\partial} - m)\psi &= V_k^{\parallel} j_k + j_0 \frac{1}{\Delta - M^2} \partial_k \dot{V}_k^{\parallel} = \\ &= \bar{\psi}^R(i \not{\partial} - m)\psi^R - V_k^{\text{R}\parallel} j_k,\end{aligned}\quad (1.30)$$

where

$$V_k^{\text{R}\parallel} = V_k^{\parallel} - \partial_k \frac{1}{\Delta - M^2} \partial_i V_i = -M^2 \frac{1}{\Delta - M^2} V_k^{\parallel}, \quad (1.31)$$

$$\psi^R = \exp \left\{ -ie \frac{1}{\Delta - M^2} \partial_i V_i \right\} \psi \quad (1.32)$$

are the radiation-type variables. It removes the linear term  $\partial_i \dot{V}_i$  in the Gauss law (1.27). If the mass  $M \neq 0$ , one can pass from the initial variables  $V_k^{\parallel}$  to the radiation ones  $V_k^{\text{R}\parallel}$  by the change

$$V_k^{\parallel} = \hat{Z} V_k^{\text{R}\parallel}, \quad \hat{Z} = \frac{M^2 - \Delta}{M^2} \quad (1.33)$$

Now the Lagrangian (1.29) goes into

$$\begin{aligned}\mathcal{L} &= \frac{1}{2} \left[ (\dot{V}_k^{\text{T}})^2 + V_k^{\text{T}} (\Delta - M^2) V_k^{\text{T}} + j_0 \frac{1}{\Delta - M^2} j_0 \right] + \bar{\psi}^R(i \not{\partial} - m)\psi^R \\ &+ \frac{1}{2} \left[ \dot{V}_k^{\text{R}\parallel} \hat{Z} \dot{V}_k^{\text{R}\parallel} + V_k^{\text{R}\parallel} (\Delta - M^2) \hat{Z} V_k^{\text{R}\parallel} \right] - V_k^{\text{T}} j_k - V_k^{\text{R}\parallel} j_k.\end{aligned}\quad (1.34)$$

The Hamiltonian corresponding to this Lagrangian can be constructed in the standard canonical way. Using the rules of the Legendre transformation and canonical conjugate momenta  $\Pi_{V_k^{\text{T}}}$ ,  $\Pi_{V_k^{\text{R}\parallel}}$ ,  $\Pi_{\psi^R}$  we obtain

$$\mathcal{H} = \frac{1}{2} \left[ \Pi_{V_k^{\text{T}}}^2 + V_k^{\text{T}} (M^2 - \Delta) V_k^{\text{T}} + j_0 \frac{1}{M^2 - \Delta} j_0 \right] - \quad (1.35)$$

$$\begin{aligned}&- \Pi_{\psi^R} \gamma_0 (i \gamma_k \partial_k + m) \psi^R \\ &+ \frac{1}{2} \left[ \Pi_{V_k^{\text{R}\parallel}} \hat{Z}^{-1} \Pi_{V_k^{\text{R}\parallel}} + V_k^{\text{R}\parallel} (M^2 - \Delta) \hat{Z} V_k^{\text{R}\parallel} \right] + \\ &+ V_k^{\text{T}} j_k + V_k^{\text{R}\parallel} j_k.\end{aligned}\quad (1.36)$$

One can be convinced [35] that the corresponding quantum system has a vacuum as a state with minimal energy and correct relativistic transformation properties.

### 1.2.3 Quantization

We start the quantization procedure from the canonical quantization by using the following equal time canonical commutation relations

(ETCCRs):

$$\left[ \hat{\Pi}_{V_k^T}, \hat{V}_k^T \right] = i \delta_{ij}^T \delta^3(\mathbf{x} - \mathbf{y}), \quad (1.37)$$

$$\left[ \hat{\Pi}_{V_k^{R||}}, \hat{V}_k^{R||} \right] = i \delta_{ij}^{||} \delta^3(\mathbf{x} - \mathbf{y}). \quad (1.38)$$

The Fock space of the theory is built by the ETCCRs

$$\left[ a_{(\lambda)}^-(\pm k), a_{(\lambda')}^+(\pm k') \right] = \delta^3(\mathbf{k} - \mathbf{k}') \delta_{(\lambda)(\lambda')}; \quad (1.39)$$

$$\{ b_{\alpha}^-(\pm k), b_{\alpha'}^+(\pm k') \} = \delta^3(\mathbf{k} - \mathbf{k}') \delta_{\alpha\alpha'}; \quad (1.40)$$

$$\{ c_{\alpha}^-(\pm k), c_{\alpha'}^+(\pm k') \} = \delta^3(\mathbf{k} - \mathbf{k}') \delta_{\alpha\alpha'}. \quad (1.41)$$

with the vacuum state  $|0\rangle$  defined by the relations

$$a_{(\lambda)}^-|0\rangle = b_{\alpha}^-|0\rangle = c_{\alpha}^-|0\rangle = 0. \quad (1.42)$$

The field operators have the Fourier decompositions in the plane wave basis

$$\begin{aligned} V_j(x) &= \int [d\mathbf{k}]_v \epsilon_j^{(\lambda)} \left[ a_{(\lambda)}^+(\omega, \mathbf{k}) e^{-i\omega t + i\mathbf{k}\mathbf{x}} + a_{(\lambda)}^-(\omega, -\mathbf{k}) e^{i\omega t - i\mathbf{k}\mathbf{x}} \right] \\ \psi(x) &= \sqrt{2m_s} \int [d\mathbf{k}]_s \left[ b_{\alpha}^+(k) u_{\alpha} e^{-i\omega t + i\mathbf{k}\mathbf{x}} + c_{\alpha}^-(k) \nu_{\alpha} e^{i\omega t - i\mathbf{k}\mathbf{x}} \right] \\ \psi^+(x) &= \sqrt{2m_s} \int [d\mathbf{k}]_s \left[ b_{\alpha}^-(k) u_{\alpha}^+ e^{i\omega t - i\mathbf{k}\mathbf{x}} + c_{\alpha}^+(-k) \nu_{\alpha}^+ e^{-i\omega t + i\mathbf{k}\mathbf{x}} \right] \end{aligned}$$

with the integral measure  $[d\mathbf{k}]_{v,s} = \frac{1}{(2\pi)^{3/2}} \frac{d^3\mathbf{k}}{\sqrt{2\omega_{v,s}(\mathbf{k})}}$  and the frequency of oscillations  $\omega_{v,s}(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m_{v,s}^2}$ . One can define the vacuum expectation values of the instantaneous products of the field operators

$$V_i(t, \vec{x}) V_j(t, \vec{y}) =: V_i(t, \vec{x}) V_j(t, \vec{y}) : + \langle V_i(t, \vec{x}) V_j(t, \vec{y}) \rangle, \quad (1.43)$$

$$\overline{\psi}_{\alpha}(t, \vec{x}) \psi_{\beta}(t, \vec{y}) =: \overline{\psi}_{\alpha}(t, \vec{x}) \psi_{\beta}(t, \vec{y}) : + \langle \overline{\psi}_{\alpha}(t, \vec{x}) \psi_{\beta}(t, \vec{y}) \rangle, \quad (1.44)$$

where

$$\langle V_i(t, \vec{x}) V_j(t, \vec{y}) \rangle = \frac{1}{(2\pi)^3} \int \frac{d^3\mathbf{k}}{2\omega_v(\mathbf{k})} \sum_{(\lambda)} \epsilon_i^{(\lambda)} \epsilon_j^{(\lambda)} e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})}, \quad (1.45)$$

$$\langle \overline{\psi}_{\alpha}(t, \vec{x}) \psi_{\beta}(t, \vec{y}) \rangle = \frac{1}{(2\pi)^3} \int \frac{d^3\mathbf{k}}{2\omega_s(\mathbf{k})} (\mathbf{k}\vec{\gamma} + m)_{\alpha\beta} e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})} \quad (1.46)$$

are the Pauli – Jordan functions.



### 1.2.4 Propagators and condensates

The vector field in the Lagrangian (1.34) is given by the formula

$$V_i^R = \left[ \delta_{ij}^T + \hat{Z}^{-1} \delta_{ij}^{\parallel} \right] V_j = V_i^T + \hat{Z}^{-1} V_i^{\parallel}. \quad (1.47)$$

Hence, the propagator of the massive vector field in radiative variables is

$$\begin{aligned} D_{ij}^R(x-y) &= \langle 0 | T V_i^R(x) V_j^R(y) | 0 \rangle = \\ &= -i \int \frac{d^4 q}{(2\pi)^4} \frac{e^{-iq \cdot (x-y)}}{q^2 - M^2 + i\epsilon} \left( \delta_{ij} - \frac{q_i q_j}{\mathbf{q}^2 + M^2} \right). \end{aligned} \quad (1.48)$$

Together with the instantaneous interaction described by the current-current term in the Lagrangian (1.34) this propagator leads to the amplitude

$$\begin{aligned} T^R &= D_{\mu\nu}^R(q) \tilde{j}^\mu \tilde{j}^\nu = \\ &= \frac{\tilde{j}_0^2}{\mathbf{q}^2 + M^2} + \left( \delta_{ij} - \frac{q_i q_j}{\mathbf{q}^2 + M^2} \right) \frac{\tilde{j}_i \tilde{j}_j}{q^2 - M^2 + i\epsilon} \end{aligned} \quad (1.49)$$

of the current-current interaction which differs from the acceptable one

$$T^L = \tilde{j}^\mu D_{\mu\nu}^L(q) \tilde{j}^\nu = -\tilde{j}^\mu \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}}{q^2 - M^2 + i\epsilon} \tilde{j}^\nu. \quad (1.50)$$

The amplitude given by Eq. (1.49) is the generalization of the radiation amplitude in QED. As it was shown in [35], the Lorentz transformations of classical radiation variables coincide with the quantum ones and they both (quantum and classical) correspond to the transition to another Lorentz frame of reference distinguished by another time-axis, where the relativistic covariant propagator takes the form

$$D_{\mu\nu}^R(q|n) = \frac{-g_{\mu\nu}}{q^2 - M^2 + i\epsilon} + \frac{n_\mu n_\nu (qn)^2 - [q_\mu - n_\mu(qn)][q_\nu - n_\nu(qn)]}{(q^2 - M^2 + i\epsilon)(M^2 + |q_\mu - n_\mu(qn)|^2)} \quad (1.51)$$

where  $n_\mu$  is determined by the external states. Remember that the conventional local field massive vector propagator takes the form (1.50)

$$D_{\mu\nu}^L(q) = -\frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}}{q^2 - M^2 + i\epsilon}. \quad (1.52)$$

In contrast to this conventional massive vector propagator the radiation-type propagator (1.51) is regular in the limit  $M \rightarrow 0$  and is well behaved for large momenta, whereas the propagator (1.52) is singular.

The radiation amplitude (1.49) can be rewritten in the alternative form

$$T^R = -\frac{1}{q^2 - M^2 + i\epsilon} \left[ \tilde{j}_\nu^2 + \frac{(\tilde{j}_i q_i)^2 - (\tilde{j}_0 q_0)^2}{\tilde{q}^2 + M^2} \right], \quad (1.53)$$

for comparison with the conventional amplitude defined by the propagator (1.52). One can find that for a massive vector field coupled to a conserved current ( $q_\mu \tilde{j}^\mu = 0$ ) the collective current-current interactions mediated by the radiation propagator (1.51) and by the conventional propagator (1.52) coincide

$$\tilde{j}^\mu D_{\mu\nu}^R \tilde{j}^\nu = \tilde{j}^\mu D_{\mu\nu}^L \tilde{j}^\nu = T^L. \quad (1.54)$$

If the current is not conserved  $\tilde{j}_0 q_0 \neq \tilde{j}_k q_k$ , the collective radiation field variables with the propagator (1.51) are inequivalent to the initial local variables with the propagator (1.52), and the amplitude (1.49). The amplitude (1.54) in the Feynman gauge is

$$T^L = -\frac{j^2}{q^2 - M^2 + i\epsilon}, \quad (1.55)$$

and corresponds to the Lagrangian

$$\mathcal{L}_F = \frac{1}{2}(\partial_\mu V_\mu)^2 - j_\mu V_\mu + \frac{1}{2}M^2 V_\mu^2 \quad (1.56)$$

In this theory the time component has a negative contribution to the energy. According to this a correctly defined vacuum state could not exist. Nevertheless, the vacuum expectation value  $\langle V_\mu(x) V_\mu(x) \rangle$  coincides with the values for two propagators (1.51) and (1.52) because in both these propagators the longitudinal part does not give a contribution if one treats them as derivatives of constant like  $\langle \partial V_\mu(x) V_\mu(x) \rangle = \partial \langle V_\mu(x) V_\mu(x) \rangle = 0$ . In this case, we have

$$\langle V_\mu(x) V_\mu(x) \rangle = -\frac{2}{(2\pi)^3} \int \frac{d^3 \mathbf{k}}{\omega_v(\mathbf{k})} = 2L_v^2(M_v), \quad (1.57)$$

$$\langle \bar{\psi}_\alpha(x) \psi_\alpha(x) \rangle = -\frac{m_s}{(2\pi)^3} \int \frac{d^3 \mathbf{k}}{\omega_s(\mathbf{k})} = m_s L_s^2(m_s), \quad (1.58)$$

where  $m_s$ ,  $M_v$  are masses of the spinor and vector fields, and  $L_{s,v}^2$  are values of the integrals.

## 1.3 Electroweak Standard Model

### 1.3.1 The SM action

The Standard Model constructed on the Yang–Mills theory [39] with the symmetry group  $SU(2) \times U(1)$  is known as the Glashow–Weinberg–Salam theory of electroweak interactions [40]. The action of the Standard Model in the electroweak sector with presence of the Higgs field

can be written in the form

$$S_{\text{SM}} = \int d^4x \mathcal{L}_{\text{SM}} = \int d^4x [\mathcal{L}_{\text{Ind}} + \mathcal{L}_{\text{Higgs}}], \quad (1.59)$$

where

$$\begin{aligned} \mathcal{L}_{\text{Ind}} = & -\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + \sum_s \bar{s}_1^R \imath \gamma^\mu \left( D_\mu^{(-)} + \imath g' B_\mu \right) s_1^R + \sum_s \bar{L}_s \imath \gamma^\mu D_\mu^{(+)} L_s, \end{aligned} \quad (1.60)$$

is the Higgs field independent part of the Lagrangian and

$$\mathcal{L}_{\text{Higgs}} = \partial_\mu \phi \partial^\mu \phi - \phi \sum_s f_s \bar{s} s + \frac{\phi^2}{4} \sum_v g_v^2 V^2 - \underbrace{\lambda [\phi^2 - C^2]^2}_{V_{\text{Higgs}}} \quad (1.61)$$

is the Higgs field dependent part. Here

$$\sum_s f_s \bar{s} s \equiv \sum_{s=s_1, s_2} f_s [\bar{s}_s R s_s L + \bar{s}_s L s_s R], \quad (1.62)$$

$$\frac{1}{4} \sum_{v=W_1, W_2, Z} g_v^2 V^2 \equiv \frac{g^2}{4} W_\mu^+ W^{-\mu} + \frac{g^2 + g'^2}{4} Z_\mu Z^\mu \quad (1.63)$$

are the mass-like terms of fermions and W-,Z-bosons coupled with the Higgs field,  $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\varepsilon_{abc}A_\mu^b A_\nu^c$  is the field strength of non-Abelian  $SU(2)$  fields and  $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$  is the field strength of Abelian  $U(1)$  (electromagnetic interaction) ones,  $D_\mu^{(\pm)} = \partial_\mu - ig\frac{\tau_a}{2}A_\mu^a \pm \frac{i}{2}g'B_\mu$  are the covariant derivatives,  $\bar{L}_s = (\bar{s}_1^L \bar{s}_2^L)$  are the fermion doublets,  $g$  and  $g'$  are the Weinberg coupling constants, and measurable gauge bosons  $W_\mu^+$ ,  $W_\mu^-$ ,  $Z_\mu$  are defined by the relations:

$$W_\mu^\pm \equiv A_\mu^1 \pm A_\mu^2 = W_\mu^1 \pm W_\mu^2, \quad (1.64)$$

$$Z_\mu \equiv -B_\mu \sin \theta_W + A_\mu^3 \cos \theta_W, \quad (1.65)$$

$$\tan \theta_W = \frac{g'}{g}, \quad (1.66)$$

where  $\theta_W$  is the Weinberg angle.

The crucial meaning has a distribution of the Higgs field  $\phi$  on the zeroth Fourier harmonic

$$\langle \phi \rangle = \frac{1}{V_0} \int d^3x \phi \quad (1.67)$$

and the nonzeroth ones  $h$ , which we will call the Higgs boson

$$\phi = \langle \phi \rangle + \frac{h}{\sqrt{2}}, \quad \int d^3x h = 0. \quad (1.68)$$

In the acceptable way,  $\langle\phi\rangle$  satisfies the particle vacuum classical equation ( $\hbar = 0$ )

$$\frac{\delta V_{\text{Higgs}}(\langle\phi\rangle)}{\delta\langle\phi\rangle} = 4\langle\phi\rangle[\langle\phi\rangle^2 - C^2] = 0 \quad (1.69)$$

that has two solutions

$$\langle\phi\rangle_1 = 0, \quad \langle\phi\rangle_2 = C \neq 0. \quad (1.70)$$

The second solution corresponds to the spontaneous vacuum symmetry breaking that determines the masses of all elementary particles

$$M_W = \frac{\langle\phi\rangle}{\sqrt{2}}g \quad (1.71)$$

$$M_Z = \frac{\langle\phi\rangle}{\sqrt{2}}\sqrt{g^2 + g'^2} \quad (1.72)$$

$$m_s = \langle\phi\rangle y_s, \quad (1.73)$$

according to the definitions of the masses of vector (v) and fermion (s) particles

$$\mathcal{L}_{\text{mass terms}} = \frac{M_v^2}{2}V_\mu V^\mu - m_s \bar{s}s. \quad (1.74)$$

### 1.3.2 Hamiltonian approach to SM

The accepted SM (1.59) is bilinear with respect to the time components of the vector fields  $V_0^K = (A_0, Z_0, W_0^+, W_0^-)$  in the “comoving frame”  $n_\mu^{\text{cf}} = (1, 0, 0, 0)$

$$S_V = \int d^4x \left[ \frac{1}{2}V_0^K \hat{L}_{00}^{KI} V_0^I + V_0^K J^K + \dots \right], \quad (1.75)$$

where  $\hat{L}_{00}^{KI}$  is the matrix of differential operators. Therefore, the Dirac approach to SM can be realized. This means that the problems of the reduction and diagonalization of the set of the Gauss laws are solvable, and the Poincaré algebra of gauge-invariant observables can be proved [35]. In any case, SM in the lowest order of perturbation theory is reduced to the sum of the Abelian massive vector fields, where Dirac’s radiation variables were considered in Section 3.

### 1.3.3 The conformal vacuum Higgs effect

The Hamiltonian approach to the Standard Model considered in [41] leads to fundamental operator quantization that allows a possibility of dynamic spontaneous symmetry breaking based on the Higgs potential (1.61), where instead of a dimensional parameter  $C$  we substitute the zeroth Fourier harmonic (1.67)

$$\mathcal{L}_{\text{Higgs}} = \partial_\mu \phi \partial^\mu \phi - \phi \sum_s f_s \bar{s}s + \frac{\phi^2}{4} \sum_v g_v^2 V^2 - \underbrace{\lambda [\phi^2 - \langle\phi\rangle^2]^2}_{V_{\text{Higgs}}}. \quad (1.76)$$

After the separation of the zeroth mode (1.68) the bilinear part of the Higgs Lagrangian takes the form

$$\mathcal{L}_{\text{Higgs}}^{\text{bilinear}} = \frac{1}{2} \partial_\mu h \partial^\mu h - \langle \phi \rangle \sum_s f_s \bar{s} s + \frac{\langle \phi \rangle^2}{4} \sum_v g_v^2 V^2 - 2\lambda \langle \phi \rangle^2 h^2. \quad (1.77)$$

In the lowest order in the coupling constant, the bilinear Lagrangian of the sum of all fields  $S^{(2)} = \sum_F S_F^{(2)}[\langle \phi \rangle]$  arises with the masses of vector (1.71), (1.72), fermion (s) (1.73) and Higgs (h) particles:

$$m_h = 2\sqrt{\lambda} \langle \phi \rangle. \quad (1.78)$$

The sum of all vacuum-vacuum transition amplitude diagrams of the theory is known as the effective Coleman – Weinberg potential [42]

$$\mathcal{V}_{eff}^{\text{conf}} = -i \text{Tr} \log < 0 | 0 >_{(\langle \phi \rangle)} = -i \text{Tr} \log \prod_F G_F^{-A_F}[\langle \phi \rangle] G_F^{A_F}[\phi_I] \quad (1.79)$$

where  $G_F^{-A_F}$  are the Green-function operators with  $A_F = 1/2$  for bosons and  $A_F = -1$  for fermions. In this case, the unit vacuum-vacuum transition amplitude  $< 0 | 0 > \Big|_{\langle \phi \rangle = \phi_I} = 1$  means that

$$\mathcal{V}_{eff}^{\text{conf}}(\phi_I) = 0, \quad (1.80)$$

where  $\phi_I$  is a solution of the variation equation

$$\begin{aligned} \partial_0^2 \langle \phi \rangle + \frac{d\mathcal{V}_{eff}^{\text{conf}}(\langle \phi \rangle)}{d\langle \phi \rangle} \Big|_{\langle \phi \rangle = \phi_I} &= \partial_0^2 \langle \phi \rangle + \\ + \sum_s f_s < \bar{s} s > - \frac{\langle \phi \rangle}{2} \sum_v g_v^2 < V^2 > + 4\lambda \langle \phi \rangle < h^2 > &= 0, \end{aligned} \quad (1.81)$$

here  $< V^2 >, < \bar{s} s >, < h^2 >$  are the condensates determined by the Green functions in [41]

$$< V^2 > = \langle V_\mu(x) V_\mu(x) \rangle = -2L_v^2(M_{Rv}), \quad (1.82)$$

$$< \bar{s} s > = \langle \bar{\psi}_\alpha(x) \psi_\alpha(x) \rangle = -m_{Rs} L_s^2(m_{Rs}); \quad (1.83)$$

$$< h^2 > = \langle h(x) h(x) \rangle = \frac{1}{2} L_h^2(m_{Rh}); \quad (1.84)$$

here  $L_p^2(m_p^2)$  are values of the integral

$$L_p^2(m_p^2) = \frac{1}{(2\pi)^3} \int \frac{d^3 \mathbf{k}}{\sqrt{m_p^2 + \mathbf{k}^2}}. \quad (1.85)$$

Finally, using the definitions of the condensates and masses (1.71), (1.72), (1.73), (1.78) we obtain the equation of motion

$$\langle\phi\rangle\partial_0^2\langle\phi\rangle = \sum_s m_s^2 L_s^2 - 2 \sum_v M_v^2 L_v^2 - \frac{1}{2} m_h^2 L_h^2. \quad (1.86)$$

In the class of constant solutions  $\partial_0^2\langle\phi\rangle \equiv 0$  this equation has two solutions

$$\langle\phi\rangle_1 = 0, \quad \langle\phi\rangle_2 = C \neq 0. \quad (1.87)$$

The nonzero solution means that there is the Gell-Mann–Oakes–Renner type relation

$$L_h^2 m_h^2 = 2 \sum_{s=s_1, s_2} L_s^2 m_s^2 - 4[2M_W^2 L_W^2 + M_Z^2 L_Z^2]. \quad (1.88)$$

If we suppose that the condensates  $L_p^2(m_{Rp}^2)$  are defined by the subtraction procedure associated with the renormalization of masses and wave functions leading to the finite value

$$\begin{aligned} L_{Rp}^2(m_{Rp}^2) &= L_p^2(m_{Rp}^2) - L_p^2(\Lambda^2) - (m_{Rp}^2 - \Lambda^2) \frac{d}{d\Lambda^2} L_p^2(\Lambda^2) = \\ &= \frac{m_{Rp}^2}{2(2\pi)^2} \log \frac{m_{Rp}^2}{e\Lambda^2}, \end{aligned} \quad (1.89)$$

where  $\Lambda$  is a subtraction constant.

In this case, the sum rule (1.88) takes the form

$$\begin{aligned} L_{Rh}^2(m_{Rh}^2) m_h^2 &= 2 \sum_{f=f_1, f_2} L_{Rf}^2(m_{Rf}^2) m_f^2 - \\ &- 4[2M_W^2 L_{RW}^2(M_{RW}^2) + M_Z^2 L_{RZ}^2(M_{RZ}^2)]. \end{aligned} \quad (1.90)$$

We substitute the experimental data by the values of masses of bosons  $M_W = 80.403 \pm 0.029$  GeV,  $M_Z = 91.1876 \pm 0.00021$  GeV [43], and t-quark  $m_t = 170.9 \pm 1.8$  GeV [44]. In the minimal SM [38], the three color t-quark dominates  $\sum_f m_f^2 \simeq 3m_t^2$  because contributions of other fermions  $\sum_{f \neq t} m_f^2 / 2m_t \sim 0.17$  GeV are very small.

In Fig. 5.1 the solution of the above equation is plotted for the range  $0.3 \text{ GeV} < \Lambda < 100 \text{ GeV}$ . One can see that the Higgs mass is the order of  $215 \div 255$  GeV and is not very sensitive to the choice of the parameter, because the dependence is logarithmic. The measurement of the mass at an experiment would provide us the proper value of  $\Lambda$  according to Fig. 5.1.

Radiative corrections to this quantity in the Standard Model are not small, first of all due to a large coupling constant of Higgs with top quark. Note that relation (1.90) in our model should be valid in all orders of the perturbation theory.

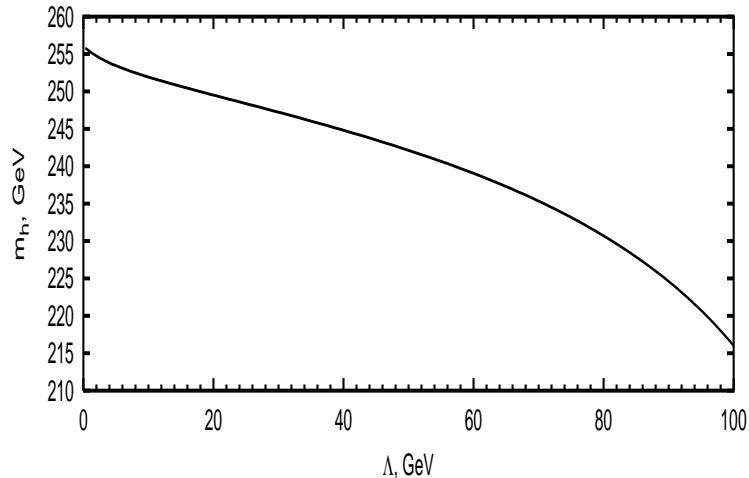


Figure 1: Value of the Higgs mass from Eq. (1.90) with the condensates defined by Eq. (1.89) as a function of  $\Lambda$ .

The choice of the parameters in the inertial Higgs potential in our model can be motivated by the cosmological reasons. Even so that the resulting Lagrangian of the model is practically the same as the one of SM, we get a prediction for value of the the Higgs boson mass to be in the range  $215 \div 255$  GeV. In this range of  $m_h$  the width of the Higgs particle is between 5 and 10 GeV. Here the main decay modes are  $W \rightarrow ZZ$  and  $H \rightarrow WW$  (since  $M_Z < m_h < 2m_t$ ), which are quite convenient for experimental studies [47]. The so-called “gold-plated” channel  $H \rightarrow 4\mu$  should allow a rather accurate measurement of  $m_h$  with at least 0.1% relative error [45]. So it is important to provide adequately precise theoretical predictions for this quantity. As concerns the production mechanism, the sub-process with gluon-gluon fusion dominates [46] for the given range of  $m_h$  and the corresponding cross section of about  $10^4$  fb provides a good possibility to discover the Higgs boson at the high-luminosity LHC machine.

In this way the potential free Higgs mechanism gives the possibility to solve the question about a consistence of the nonzero vacuum value of the scalar field with the zero vacuum cosmological energy as a consequence of the unit vacuum-vacuum transition amplitude. The inertial motion of a scalar field corresponds to the dominance of the most singular rigid state at the epoch of the intensive vacuum creation of the primordial bosons [15].

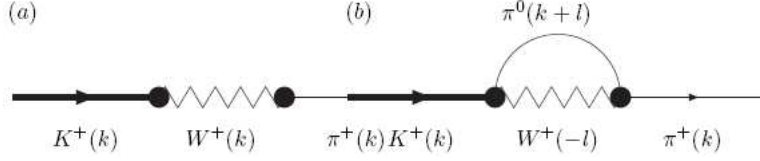


Figure 2: Axial (a) and vector (b) current contribution into  $K^+ \rightarrow \pi^+$  transition

### 1.3.4 The static interaction mechanism of the enhancement of the $\Delta T = 1/2$ transitions

Let us consider the  $K^+ \rightarrow \pi^+$  transition amplitude

$$\left\langle \pi^+ \left| -i \int d^4x d^4y J^\mu(x) D_{\mu\nu}^W J^\nu(y) \right| K^+ \right\rangle = i(2\pi)^4 \delta^4(k-p) G_{EW} \Sigma(k^2)$$

where  $D_{\mu\nu}^W \equiv D_{\mu\nu}^W(x-y)$  in the first order of the EW perturbation theory in the Fermi coupling constant

$$G_{EW} = \frac{\sin \theta_C \cos \theta_C}{8M_W^2} \frac{e^2}{\sin^2 \theta_W} \equiv \sin \theta_C \cos \theta_C \frac{G_F}{\sqrt{2}}, \quad (1.91)$$

comparing two different W-boson field propagators, the accepted Lorentz (L) propagator (1.52) and the radiation (R) propagator (1.51). These propagators give the expressions corresponding to the diagrams in Fig. 2

$$\begin{aligned} \Sigma^R(k^2) &= 2F_\pi^2 k^2 - 2i \int \frac{d^4q M_W^2}{(2\pi)^4} \frac{k^2 + (k_0 + q_0)^2}{(|\vec{q}|^2 + M_W^2)[(k+q)^2 - m_\pi^2 + i\epsilon]}, \\ \Sigma^L(k^2) &= 2F_\pi^2 k^2 + 2i \int \frac{d^4q M_W^2}{(2\pi)^4} \frac{(2k_\mu + q_\mu) D_{\mu\nu}^L(-q) (2k_\nu + q_\nu)}{(k+q)^2 - m_\pi^2 + i\epsilon}. \end{aligned}$$

The versions R and L coincide in the case of the axial contribution corresponding to the first diagram in Fig. 2, and they both reduce to the static interaction contribution because

$$k^\mu k^\nu D_{\mu\nu}^F(k) \equiv k^\mu k^\nu D_{\mu\nu}^R(k) = \frac{k_0^2}{M_W^2}. \quad (1.92)$$

However, in the case of the vector contribution corresponding to the second diagram in Fig. 2 the radiation version differs from the Lorentz gauge version (1.52).

In contrast to the Lorentz gauge version (1.52), two radiation variable diagrams in Fig. 2 in the rest kaon frame  $k_\mu = (k_0, 0, 0, 0)$  are reduced to the static interaction contribution

$$i(2\pi)^4 \delta^4(k-p) G_{EW} \Sigma^R(k^2) = \left\langle \pi^+ \left| -i \int d^4x \frac{J_0^2(x)}{\Delta - M_W^2} \right| K^+ \right\rangle$$



with the normal ordering of the pion fields which are at their mass-shell, so that

$$\Sigma^R(k^2) = 2k^2 F_\pi^2 \left[ 1 + \frac{M_W^2}{F_\pi^2 (2\pi)^3} \int \frac{d^3 l}{2E_\pi(\vec{l})} \frac{1}{M_W^2 + \vec{l}^2} \right] \equiv 2k^2 F_\pi^2 g_8.$$

Here  $E_\pi(\vec{l}) = \sqrt{m_\pi^2 + \vec{l}^2}$  is the energy of  $\pi$ -meson and  $g_8$  is the parameter of the enhancement of the probability of the axial  $K^+ \rightarrow \pi^+$  transition. The pion mass-shell justifies the application of the low-energy ChPT [48, 49], where the summation of the chiral series can be considered here as the meson form factors [50, 51, 52]

$$\int \frac{d^3 l}{2E_\pi(\vec{l})} \rightarrow \int \frac{d^3 l f_K^V(-(\vec{l})^2) f_\pi^V(-(\vec{l})^2)}{2E_\pi(\vec{l})} \quad (1.93)$$

Using the covariant perturbation theory [53] developed as the series

$$\underline{J}_\mu^k(\gamma \oplus \xi) = \underline{J}_\mu^k(\xi) + F_\pi^2 \partial_\mu \gamma^k + \gamma^i f_{ijk} \underline{J}_\mu^j(\xi) + O(\gamma^2) \quad (1.94)$$

with respect to quantum fields  $\gamma$  added to  $\xi$  as the product  $e^{i\gamma} e^{i\xi} \equiv e^{i(\gamma \oplus \xi)}$ , one can see that the normal ordering

$$< 0 | \gamma^i(x) \gamma^{i'}(y) | 0 > = \delta^{ii'} N(\vec{z}), \quad N(\vec{z}) = \int \frac{d^3 l e^{i\vec{l} \cdot \vec{z}}}{(2\pi)^3 2E_\pi(\vec{l})}, \quad (1.95)$$

where  $\vec{z} = \vec{x} - \vec{y}$ , in the product of the currents  $\underline{J}_\mu^k(\gamma \oplus \xi)$ , leads to an effective Lagrangian with the rule  $\triangle T = 1/2$

$$M_W^2 \int d^3 z g_8(z) \frac{e^{-M_W |\vec{z}|}}{4\pi |\vec{z}|} \left[ \underline{J}_\mu^j(x) \underline{J}_\mu^{j'}(z+x) (f_{ij1} + i f_{ij2}) \right. \\ \left. \times (f_{i'j'4} - i f_{i'j'5}) \delta^{ii'} + h.c. \right], \quad (1.96)$$

where

$$g_8(|z|) = 1 + \sum_{I \geq 1} c^I N^I(\vec{z}) \quad (1.97)$$

is a series over the multiparticle intermediate states known as the Volkov superpropagator [49, 54]. In the limit  $M_W \rightarrow \infty$ , in the lowest order with respect to  $M_W$ , the dependence of  $g_8(|\vec{z}|)$  and the currents on  $\vec{z}$  disappears in the integral of the type of

$$M_W^2 \int d^3 z \frac{g_8(|\vec{z}|) e^{-M_W |\vec{z}|}}{4\pi |\vec{z}|} = \int_0^\infty dr r e^{-r} g_8(r/M_W) \simeq g_8(0). \quad (1.98)$$

In the next order, the amplitudes  $K^0(\bar{K}^0) \rightarrow \pi^0$  arise. Finally, we get the effective Lagrangians [55]

$$\begin{aligned} \mathcal{L}_{(\Delta T=\frac{1}{2})} = & \frac{G_F}{\sqrt{2}} g_8(0) \cos \theta_C \sin \theta_C \left[ (\underline{J}_\mu^1 + i \underline{J}_\mu^2)(\underline{J}_\mu^4 - i \underline{J}_\mu^5) - \right. \\ & \left. \left( \underline{J}_\mu^3 + \frac{1}{\sqrt{3}} \underline{J}_\mu^8 \right) (\underline{J}_\mu^6 - i \underline{J}_\mu^7) + h.c. \right], \end{aligned} \quad (1.99)$$

$$\mathcal{L}_{(\Delta T=\frac{3}{2})} = \frac{G_F}{\sqrt{2}} \cos \theta_C \sin \theta_C \left[ \left( \underline{J}_\mu^3 + \frac{1}{\sqrt{3}} \underline{J}_\mu^8 \right) (\underline{J}_\mu^6 - i \underline{J}_\mu^7) + h.c. \right].$$

This result shows that the enhancement can be explained by static vector interaction that increases the  $K^+ \rightarrow \pi^+$  transition by a factor of  $g_8 = g_8(0)$ , and yields a new term describing the  $K^0 \rightarrow \pi^0$  transition proportional to  $g_8 - 1$ .

This Lagrangian with the fit parameter  $g_8 = 5$  describes the nonleptonic decays in satisfactory agreement with experimental data [49, 55, 56]. Thus, for normal ordering of the weak static interaction in the Hamiltonian SM can explain the rule  $\Delta T = 1/2$  and universal factor  $g_8$ .

On the other hand, contact character of weak static interaction in the Hamiltonian SM excludes all retarded diagram contributions in the effective Chiral Perturbation Theory considered in [57] that destruct the form factor structure of the kaon radiative decay rates with the amplitude

$$T_{(K^+ \rightarrow \pi^+ l^+ l^-)} = g_8 t(q^2) 2F_\pi^2 \sin \theta_C \cos \theta_C \frac{G_F}{\sqrt{2}} \frac{(k_\mu + p_\mu)}{q^2} \bar{l} \gamma_\mu l$$

where  $q^2 = (k - p)^2$ , and

$$t(q^2) = \frac{f_K^A(q^2) + f_\pi^A(q^2)}{2} - f_\pi^V(q^2) + \left[ f_K^V(q^2) - f_\pi^V(q^2) \right] \frac{m_\pi^2}{M_K^2 - m_\pi^2},$$

and  $f_K^V \simeq f_\pi^V(q^2) = 1 + M_\rho^{-2} q^2 + \dots$ ,  $f_K^A(q^2) \simeq f_\pi^A(q^2) = 1 + M_a^{-2} q^2 + \dots$  are form factors determined by the masses of the nearest resonances for meson – gamma – meson vertex.

Therefore, the static interaction mechanism of the enhancement of the  $\Delta T = 1/2$  transitions predicts [56] that the meson form factor resonance parameters explain the experimental values of rates of the radiation kaon decays  $K^+ \rightarrow \pi^+ e^+ e^- (\mu^+ \mu^-)$ . Actually, substituting the PDG data on the resonance masses  $M_\rho = 775.8$  MeV,  $1^+(1^{--}) = I^G(J^{PC})$  and meson – gamma – W-boson one  $M_a = 984.7$  MeV,  $1^-(0^{++}) = I^G(J^{PC})$  into the decay amplitudes one can obtain the decay branching fractions [56]

$$\begin{aligned} \text{Br}(K^+ \rightarrow \pi^+ e^+ e^-) &= 2.93 \times 10^{-7}, \quad [2.88 \pm 0.13 \times 10^{-7}]_{\text{PDG}} \\ \text{Br}(K^+ \rightarrow \pi^+ \mu^+ \mu^-) &= 0.73 \times 10^{-7}, \quad [0.81 \pm 0.14 \times 10^{-7}]_{\text{PDG}} \end{aligned}$$

in satisfactory agreement with experimental data [58, 43]. Thus, the off-mass-shell kaon-pion transition in the radiation weak kaon decays can be a good probe of the weak static interaction revealed by the radiation propagator (1.51) of the Hamiltonian presentation of SM.

## 1.4 Summary

*Physical consequences of the Hamiltonian approach to the Standard Model are the weak static interactions, like the Coulomb static interaction is a consequence of the Hamiltonian approach in QED.* The static interactions can be omitted if we restrict ourselves to the scattering processes of elementary particles where static interactions are not important. However, the static poles play a crucial role in the mass-shell phenomena of the bound state type, spontaneous symmetry breaking, kaon - pion transition in the weak decays, etc. *Static interactions follow from the spectrality principle that means existence of a vacuum defined as a state with the minimal energy.* We discussed physical effects testifying to the static interactions omitted by the accepted version of SM.

One of these effects is revealed by the loop meson diagrams in the low-energy weak static interaction. These diagrams lead to the enhancement coefficient  $g_8$  in weak kaon decays and the rule  $\Delta T = \frac{1}{2}$ . The loop pion diagrams in the Chiral Perturbation Theory [49] in the framework of the Hamiltonian approach with the weak static interaction lead to a definite relation of the vector form factor with the differential radiation kaon decay rates in agreement with the present day PDG data [56], in contrast to the acceptable renormalization group analysis based on the Lorentz gauge formulation omitting weak static interaction [57], where loop pion diagrams destroy the above-mentioned relation of the vector form factor with the differential radiation kaon decay rates. *Therefore, the radiation kaon decays can be a good probe of the weak static interaction.*

We considered the consequence of the spontaneous symmetry breaking in the Standard Model, where the parameter of the Higgs potential is replaced by the initial data of the zeroth Fourier harmonic. In this case, the Hamiltonian approach and its operator fundamental formulation immediately lead to the effective quantum potential predicting the mass of Higgs particle.

## 2 Hamiltonian General Relativity

The statement of the problem given at the beginning is to unify SM and GR on equal footing using as a basis the Hamiltonian approach to both these theories, in order to describe the Universe in its comoving frame. The Hamiltonian approach to GR is well known. It is the Dirac – ADM constrained method [13] formulated for infinite space-time in a definite frame of reference, where the observable time is distinguished and the observable space is foliated. This Hamiltonian comoving frame of the Universe can be identified with the frame of the Cosmic Microwave Background (CMB) as the evidence of the Early Universe creation.

The present-day measurement of the dipole component of CMB radiation temperature  $T_0(\theta) = T_0[1 + (\beta/c) \cos \theta]$ , where  $\beta = 390 \pm 30$  km/s, [1] testifies to a motion of an Earth observer to the Leo with the velocity  $|\vec{v}| = 390 \pm 30$  km/s with respect to CMB, where 30 km/s rejects the copernican annual motion of the Earth around the Sun, and 390 km/s to the Leo is treated as the parameter of the Lorentz transformation from the the Earth frame to the CMB frame.

This relativistic treatment of the observational data in the context of the Hamiltonian approach produces the definite questions to the GR and the modern cosmological models destined for description of the processes of origin of the Universe and its evolution:

1. How the CMB *inertial frame* can be separated from the general coordinate transformations?
2. How the cosmic evolution can be separated from the dynamics of the local scalar component in the CMB reference frame?
3. What is the version of the canonical approach to the General Relativity and the Standard Model in the finite space-time, because the observable Universe in the finite space and has a finite life-time?

In this Section, we discuss possible responses to these issues that follow from the principles of General Relativity and Quantum Field Theory.

### 2.1 Canonical General Relativity

#### 2.1.1 The Fock separation of the frame transformations from diffeomorphisms

Recall that the Einstein–Hilbert theory is given by two fundamental quantities; they are a *geometric interval*

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (2.1)$$

and the *dynamic* Hilbert action

$$S_{\text{GR}} = \int d^4x \sqrt{-g} \left[ -\frac{\varphi_0^2}{6} R(g) \right] \quad (2.2)$$

where  $\varphi_0^2 = \frac{3}{8\pi} M_{\text{Planck}}^2 = \frac{3}{8\pi G}$ ,  $G$  is the Newton constant in units ( $\hbar = c = 1$ ).

Quantities (2.1) and (2.2) are invariant with respect to action diffeomorphisms

$$x^\mu \rightarrow \tilde{x}^\mu = \tilde{x}^\mu(x^0, x^1, x^2, x^3), \quad (2.3)$$

Separation of the diffeomorphisms from the Lorentz transformations in GR is fulfilled by linear invariant forms  $\omega_{(\alpha)}(x^\mu) \rightarrow \omega_{(\alpha)}(\tilde{x}^\mu) = \omega_{(\alpha)}(x^\mu)$  [12]

$$ds^2 \equiv \omega_{(\alpha)}\omega_{(\alpha)} = \omega_{(0)}\omega_{(0)} - \omega_{(1)}\omega_{(1)} - \omega_{(2)}\omega_{(2)} - \omega_{(3)}\omega_{(3)}, \quad (2.4)$$

where  $\omega_{(\alpha)}$  are diffeo-invariants. These forms are treated as components of an orthogonal reference simplex with the following Lorentz transformations:

$$\omega_{(\alpha)} \rightarrow \overline{\omega}_{(\alpha)} = \overline{\omega}_{(\alpha)} = L_{(\alpha)(\beta)}\omega_{(\beta)}. \quad (2.5)$$

There is an essential difference between diffeomorphisms (2.3) and the Lorentz transformations (2.5). Namely, the parameters of the Lorentz transformations (2.5) are measurable quantities, while the parameters of diffeomorphisms (2.3) are unmeasurable one. Especially, the simplex components  $\omega_{(\alpha)}$  in the Earth frame moving with respect to Cosmic Microwave Background (CMB) radiation with the velocity  $|\vec{v}| = 390$  km/s to the Leo are connected with the simplex components in the CMB frame  $\overline{\omega}$  by the following formulae:

$$\begin{aligned} \overline{\omega}_{(0)} &= \frac{1}{\sqrt{1-\vec{v}^2}} [\omega_{(0)} - v_{(c)}\omega_{(c)}], \\ \overline{\omega}_{(b)} &= \frac{1}{\sqrt{1-\vec{v}^2}} [\omega_{(b)} - v_{(b)}\omega_{(0)}], \end{aligned} \quad (2.6)$$

where the velocities  $\vec{v}$  are measured [1] by the the modulus of the dipole component of CMB temperature  $T_0(\theta) = T_0[1 + (\beta/c)\cos\theta]$  and its direction in space<sup>1</sup>.

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<sup>1</sup>Frame transformations invariance of action means that there are integrals of motion (1st Noether theorem [27]), while diffeoinvariance of action leads to the Gauss type constraints between the integrals of motion (2nd Noether theorem [27]). These constraints are derived in a *specific reference frame to the initial data*. The constraints mean that only a part of metric components becomes *degrees of freedom* with the initial data. Another part corresponds to the diffeo-invariant *static potentials* that does not have initial data because their equations contain the Beltrami-Laplace operator. The third part of metric components after the resolution of constraints becomes diffeo-invariant non-dynamical variables that can be excluded by the gauge-constraints [13] like the longitudinal fields in QED [30].

### 2.1.2 The Dirac – ADM approach to GR

The problem of specific frame destined for description of evolution of the Universe in GR was formulated by Dirac and Arnovitt, Deser and Misner [13] as 3+1 foliated space-time (see also [14]). This foliation can be rewritten in terms of the Fock simplex components as follows:

$$\omega_{(0)} = \psi^6 N_d dx^0, \quad \omega_{(b)} = \psi^2 \mathbf{e}_{(b)i} (dx^i + N^i dx^0), \quad (2.7)$$

where triads  $\mathbf{e}_{(a)i}$  form the spatial metrics with  $\det |\mathbf{e}| = 1$ ,  $N_d$  is the Dirac lapse function,  $N^k$  is the shift vector, and  $\psi$  is a determinant of the spatial metric.

The Hilbert action (2.2) in terms of the Dirac – ADM variables (2.7) is as follows:

$$\begin{aligned} S_{\text{GR}} &= - \int d^4x \sqrt{-g} \frac{\varphi_0^2}{6} {}^{(4)}R(g) = \\ &= \int d^4x (\mathcal{K}[\varphi_0|g] - \mathcal{P}[\varphi_0|g] + \mathcal{S}[\varphi_0|g]), \end{aligned} \quad (2.8)$$

where

$$\mathcal{K}[\varphi_0|e] = N_d \varphi_0^2 \left( -4v_\psi^2 + \frac{v_{(ab)}^2}{6} \right), \quad (2.9)$$

$$\mathcal{P}[\varphi_0|e] = \frac{N_d \varphi_0^2 \psi^7}{6} \left( {}^{(3)}R(\mathbf{e})\psi + 8\Delta\psi \right), \quad (2.10)$$

$$\mathcal{S}[\varphi_0|e] = 2\varphi_0^2 [\partial_0 v_\psi - \partial_l (N^l v_\psi)] - \frac{\varphi_0^2}{3} \partial_j [\psi^2 \partial^j (\psi^6 N_d)] \quad (2.11)$$

are the kinetic and potential terms, respectively,

$$v_\psi = \frac{1}{N_d} \left[ (\partial_0 - N^l \partial_l) \log \psi - \frac{1}{6} \partial_l N^l \right], \quad (2.12)$$

$$v_{(ab)} = \frac{1}{2} \left( \mathbf{e}_{(a)i} v_{(b)}^i + \mathbf{e}_{(b)i} v_{(a)}^i \right), \quad (2.13)$$

$$v_{(a)i} = \frac{1}{N_d} \left[ (\partial_0 - N^l \partial_l) \mathbf{e}_{(a)i} + \frac{1}{3} \mathbf{e}_{(a)i} \partial_l N^l - \mathbf{e}_{(a)l} \partial_i N^l \right] \quad (2.14)$$

are velocities of the metric components,  $\Delta\psi = \partial_i (\mathbf{e}_{(a)}^i \mathbf{e}_{(a)}^j \partial_j \psi)$  is the covariant Beltrami–Laplace operator,  ${}^{(3)}R(\mathbf{e})$  is a three-dimensional curvature expressed in terms of triads  $\mathbf{e}_{(a)i}$ :

$${}^{(3)}R(\mathbf{e}) = -2\partial_i [\mathbf{e}_{(b)}^i \sigma_{(c)|(b)(c)}] - \sigma_{(c)|(b)(c)} \sigma_{(a)|(b)(a)} + \sigma_{(c)|(d)(f)} \sigma_{(f)|(d)(c)}.$$

Here

$$\sigma_{(a)|(b)(c)} = \mathbf{e}_{(c)}^j \nabla_i \mathbf{e}_{(a)k} \mathbf{e}_{(b)}^k = \frac{1}{2} \mathbf{e}_{(a)j} \left[ \partial_{(b)} \mathbf{e}_{(c)}^j - \partial_{(c)} \mathbf{e}_{(b)}^j \right] \quad (2.15)$$

are the coefficients of the spin-connection (see [29]),

$$\nabla_i \mathbf{e}_{(a)j} = \partial_i \mathbf{e}_{(a)j} - \Gamma_{ij}^k \mathbf{e}_{(a)k}, \quad \Gamma_{ij}^k = \frac{1}{2} \mathbf{e}_{(b)}^k (\partial_i \mathbf{e}_{(b)j} + \partial_j \mathbf{e}_{(b)i}) \quad (2.16)$$

are covariant derivatives. The canonical conjugated momenta are

$$p_\psi = \frac{\partial \mathcal{K}[\varphi_0|e]}{\partial(\partial_0 \ln \psi)} = -8\varphi_0^2 v, \quad (2.17)$$

$$p_{(b)}^i = \frac{\partial \mathcal{K}[\varphi_0|e]}{\partial(\partial_0 \mathbf{e}_{(a)i})} = \frac{\varphi^2}{3} \mathbf{e}_{(a)}^i v_{(ab)}. \quad (2.18)$$

The Hamiltonian action takes the form [28, 29]

$$S_{\text{GR}} = \int d^4x \left[ \sum_{F=e, \log \psi, Q} P_F \partial_0 F - \mathcal{H}_d \right] \quad (2.19)$$

where

$$\mathcal{H}_d = N_d T_d + N_{(b)} T_{(b)}^0 + \lambda_0 p_\psi + \lambda_{(a)} \partial_k \mathbf{e}_{(a)}^k \quad (2.20)$$

is the sum of constraints with the Lagrangian multipliers  $N_d$ ,  $N_{(b)} = \mathbf{e}_{k(b)} N^k$ ,  $\lambda_0$ ,  $\lambda_{(a)}$ , including the additional (second class) Dirac gauge conditions – the local transverse  $\partial_k \mathbf{e}_{(a)}^k = 0$  and the minimal 3-dimensional hyper-surface too

$$p_{\tilde{\psi}} = 0 \rightarrow (\partial_0 - N^l \partial_l) \log \tilde{\psi} = \frac{1}{6} \partial_l N^l, \quad (2.21)$$

and three first class constraints

$$\begin{aligned} T_{(a)}^0 = -e_{(a)}^l \frac{\delta S}{\delta N^l} &= -p_\psi \partial_{(a)} \psi + \frac{1}{6} \partial_{(a)} (p_\psi \psi) + \\ &+ 2p_{(b)(c)} \gamma_{(b)|(a)(c)} - \partial_{(b)} p_{(b)(a)} + T_{(a)\text{m}}^0 = 0 \end{aligned} \quad (2.22)$$

are the components of the total energy-momentum tensor  $T_{(a)}^0 = -\frac{\delta S}{\delta N_k} \mathbf{e}_{k(a)}$  (we included here the matter field contribution  $T_{(a)\text{m}}^0$  considered in Appendix C using as an example a massive electrodynamics), and the first class energy constraint

$$T_d[\varphi_0|\psi] = -\frac{\delta S}{\delta N_d} = \frac{4\varphi_0^2}{3} \psi^7 \Delta \psi + \sum_I \psi^I \mathcal{T}_I = 0, \quad (2.23)$$

here  $\Delta \psi \equiv \partial_{(b)} \partial_{(b)} \psi$  is the Beltrami–Laplace operator,  $\partial_{(a)} = \mathbf{e}_{(a)}^k \partial_k$ , and  $\mathcal{T}_I$  is partial energy density

$$\mathcal{T}_{I=0} = \frac{6p_{(ab)}p_{(ab)}}{\varphi_0^2} - \frac{16}{\varphi_0^2} p_\psi^2 \quad (2.24)$$

$$\mathcal{T}_{I=8} = \frac{\varphi_0^2}{6} R^{(3)}(\mathbf{e}), \quad (2.25)$$

here  $p_{(ab)} = \frac{1}{2}(\mathbf{e}_{(a)}^i \tilde{p}_{(b)i} + \mathbf{e}_{(b)}^i p_{(a)i})$  marked by the index  $I$ .

The Newton law is determined by the energy constraints  $T_d = 0$  (2.23) and the equation of motion of the spatial determinant takes the form

$$\begin{aligned} T_\psi[\varphi_0|\psi] &= -\psi \frac{\delta S}{\delta \psi} \equiv (\partial_0 - N^l \partial_l) p_\psi + \\ &+ \frac{4\varphi_0^2}{3} [7N_d \psi^7 \Delta \psi + \psi \Delta (N_d \psi^7)] + N_d \sum_I I \psi^I \mathcal{T}_I = 0. \end{aligned} \quad (2.26)$$

It is not embarrassing to check that in the region of empty space, where two dynamic variables are absent  $\mathbf{e}_{(a)k} = \delta_{(a)k}$  (i.e.  $\mathcal{T}_I = 0$ ), one can get the Schwarzschild-type solution of these equations in the form

$$\Delta \psi = 0, \quad \Delta [N_d \psi^7] = 0 \quad \rightarrow \quad \psi = 1 + \frac{r_g}{r}, \quad [N_d \psi^7] = 1 - \frac{r_g}{r}, \quad N^k = 0,$$

where  $r_g$  is the constant of the integration given by the boundary conditions that take into account massive fields and sources.

### 2.1.3 The Lichnerowicz variables and cosmological models

In the general case of massive electrodynamics considered in detail in Appendix C, the dependence of the energy momentum tensor (2.23) on the spatial determinant *potential*  $\psi$  is completely determined by the Lichnerowicz (L) transformation to the conformal-invariant variables

$$\omega_{(\mu)} = \psi^2 \omega_{(\mu)}^{(L)}, \quad (2.27)$$

$$g_{\mu\nu} = \psi^4 g_{\mu\nu}^{(L)}, \quad (2.28)$$

$$F^{(n)} = \psi^{2n} F_{(L)}^{(n)}, \quad (2.29)$$

where  $F^{(n)}$  is any field with the one of conformal weights  $(n)$ :  $n_{\text{scalar}} = -1$ ,  $n_{\text{spinor}} = -3/2$ ,  $n_{\text{vector}} = 0$ , and  $n_{\text{tensor}} = 2$ . In the case, the index  $I$  in the energy momentum tensor (2.23)  $\sum_I \psi^I \mathcal{T}_I$  runs a set of values  $I=0$  (stiff), 4 (radiation), 6 (mass), 8 (curvature)  $I = 12$  ( $\Lambda$ -term) in correspondence with a type of matter field contributions.

This  $\psi$ -independence of L-variables is compatible with the cosmological dependence of the energy density on the scale factor  $a$  in the homogeneous approximation

$$\psi^2 \simeq a(\eta), \quad (2.30)$$

$$ds^2 = a^2(\eta)[(d\eta)^2 - (dx^k)^2] \quad (2.31)$$

where the energy constraint (2.23) takes the form

$$\varphi_0^2 a'^2 = \sum_I a^{-2+I/2} \langle \mathcal{T}_I \rangle; \quad (2.32)$$



here

$$\langle T \rangle = \frac{1}{V_0} \int d^3x T \quad (2.33)$$

is averaging over the finite volume of the coordinate space  $V_0 = \int d^3x$ .

The Newton law (2.27) is compatible with the cosmological approximation (2.30) if the spacial determinant variable takes the form of a product of two factors

$$\psi^2 = a(\eta) \tilde{\psi}^2. \quad (2.34)$$

This means that the logarithm of the spacial determinant can be given as the sum of the zeroth Fourier harmonic and nonzero ones

$$\log \psi^2(x^0, x^k) = \log a(x^0) + \log \tilde{\psi}^2(x^0, x^k), \quad (2.35)$$

with the additional constraints

$$\int d^3x \log \tilde{\psi} = \int d^3x [\log \psi - \langle \log \psi \rangle] \equiv 0, \quad (2.36)$$

where  $V_0 = \int d^3x < \infty$  is the finite Lichnerowicz volume.

This presentation of the spacial determinant variable (2.34) is well known as the Lifshits cosmological perturbation theory [59, 60].

The question arises about the consistence of this cosmological perturbation theory [59, 60] defined in the finite space-time of observable coordinate space and conformal time with the Dirac – ADM Hamiltonian [13] approach proposed for infinite space-time.

How the Hubble evolution can be included into the canonical GR? and How the Dirac – ADM Hamiltonian formalism can be generalized for finite space-time in order to give the Hamiltonian version of cosmological perturbation theory? The responses to these questions were given in [28, 15] using the exact solution of the energy constraint in accord with the group of the diffeomorphisms of the Dirac – ADM foliation and second Nöther theorem.

#### 2.1.4 Global energy constraint and dimension of diffeomorphisms ( $3L + 1G \neq 4L$ )

The Dirac – ADM approach to the Einstein–Hilbert theory [30] states that five components  $\psi, N_d, N^k$  are treated as *potentials* satisfying the Laplace type equations in curved space without the initial data, three components are excluded by the gauge constraints  $\partial_k \mathbf{e}_{(b)}^k = 0$ , and only two rest transverse gravitons are considered as independent *degrees of freedom* satisfying the d’Alambert type equations with the initial data. This Dirac – ADM classification is not compatible with the group of general coordinate transformations that conserves a family of constant coordinate time hypersurfaces  $x^0 = \text{const}$ . The group

of these transformations, known as *kinematic* subgroup [14], contains only homogeneous reparameterizations of the coordinate evolution parameter ( $x^0$ ) and three local transformations of the spatial coordinates:

$$\begin{bmatrix} x^0 \\ x^i \end{bmatrix} \rightarrow \begin{bmatrix} \tilde{x}^0(x^0) \\ \tilde{x}^i(x^0, x^i) \end{bmatrix} \quad (2.37)$$

This means that dimension of the kinematic subgroup of diffeomorphisms (three local functions and one global one) does not coincide with the dimension of the constraints in the canonical approach to the classical theory of gravitation with remove four local variables (the law  $3L + 1G \neq 4L$ ). In accord with the second Nöther theorem, the dimension of the diffeomorphism group  $3L + 1G$  determines the dimension of manifold of canonical momenta that can be removed by the first class constraints. This means that the energy constraint can remove the zeroth mode (i.e. cosmological scale factor) and the zeroth mode momentum from the phase space. However, it does not mean that both the scale factor and its momentum are removed from the set of physical quantities. They become the evolution parameter in the field space of events and the event-energy.

Recall that according to the definition of all measurable quantities as diffeo-invariants [30], in finite space-time the non diffeo-invariant quantity (2.37) ( $x^0$ ) is not measurable. Wheeler and DeWitt [11] drew attention to that in this case evolution of a universe in GR is in full analogy with a relativistic particle given by the action

$$\begin{aligned} \tilde{S}_{\text{SR}}[X^0|X^k] &= -\frac{m}{2} \int d\tau \frac{1}{e_p} \left[ \left( \frac{dX^0}{d\tau} \right)^2 - \left( \frac{dX^k}{d\tau} \right)^2 + e_p^2 \right] = \\ &= \int d\tau \left[ -P_\mu \frac{dX^\mu}{d\tau} + \frac{e_p}{2m} (P_\mu^2 - m^2) \right] \end{aligned} \quad (2.38)$$

in the Minkowski space of events  $[X^0|X^k]$  and the interval  $ds = e_p d\tau$ , because both the actions (2.38) in SR and (1.68) in GR are invariant with respect to reparameterizations of the coordinate evolution parameters  $\tau \rightarrow \tilde{\tau} = \tilde{\tau}(\tau)$  and  $x^0 \rightarrow \tilde{x}^0 = \tilde{x}^0(x^0)$ , respectively, see Table 2.1.6.

In any relativistic theory given by an action and a geometrical interval [4] there are two diffeo-invariant time-like parameters: the diffeo-invariant geometrical proper time interval (g-time)  $e_p d\tau = ds$  and the one of dynamical variables  $X^0$  in the *space of events*  $[X^0|X^k]$  (d-time). Thus, in accord with the cosmological perturbation theory [59], there is a possibility to identify the dynamic evolution parameter  $a$  in the field space of events with the zeroth Fourier harmonic of the metric scalar component logarithm if the Hamiltonian formalism in finite volume is consistent with the cosmological perturbation theory.

Let us consider the Hamiltonian formalism in finite volume and find a condition of this consistence.

### 2.1.5 The separation of the zeroth mode in finite space

Reparametrizations of the coordinate evolution parameter ( $x^0$ ) mean that in finite space-time the quantity ( $x^0$ ) is not observable, and one should distinguish a diffeo-invariant homogeneous “*time-like variable*”. Modern observational data in astrophysics and cosmology [24, 60] are the irrefutable arguments in favor of identification of such a diffeo-invariant homogeneous “*evolution parameter*” with the cosmological scale factor  $a(x_0)$  introduced by the scale transformation of the metrics  $g_{\mu\nu} = a^2(x^0)\tilde{g}_{\mu\nu}$  and any field  $F^{(n)}$  with the conformal weight ( $n$ ):

$$F^{(n)} = a^n(x_0)\tilde{F}^{(n)}. \quad (2.39)$$

In particular, the curvature

$$\sqrt{-g}R(g) = a^2\sqrt{-\tilde{g}}R(\tilde{g}) - 6a\partial_0 \left[ \partial_0 a \sqrt{-\tilde{g}} \tilde{g}^{00} \right] \quad (2.40)$$

can be expressed in terms of the new lapse function  $\tilde{N}_d$  and spatial determinant  $\tilde{\psi}$  in the Fock simplex (2.7)

$$\tilde{N}_d = [\sqrt{-\tilde{g}} \tilde{g}^{00}]^{-1} = a^2 N_d, \quad \tilde{\psi} = (\sqrt{a})^{-1} \psi. \quad (2.41)$$

In order to keep the number of variables, we identify  $\log \sqrt{a}$  with the spatial volume “averaging” of  $\log \psi$

$$\log \sqrt{a} = \langle \log \psi \rangle \equiv \frac{1}{V_0} \int d^3x \log \psi. \quad (2.42)$$

After the separation of the zeroth mode the action (2.8) takes the form (2.7) as follows:

$$S_{\text{GR}}[\varphi_0|\psi] = S_{\text{GR}}[\varphi|\tilde{\psi}] + S_{\text{int}} + S_0, \quad (2.43)$$

where

$$S_{\text{GR}}[\varphi|\tilde{\psi}] = \int d^4x (\mathcal{K}[\varphi|g] - \mathcal{P}[\varphi|g] + \mathcal{S}[\varphi|g]) \quad (2.44)$$

is the action  $S_{\text{GR}}[\varphi_0|\psi]$  with the change  $[\varphi_0|\psi] \rightarrow [\varphi|\tilde{\psi}]$

$$\varphi = \varphi_0 a \quad (2.45)$$

$$v_{\tilde{\psi}} = \frac{1}{\tilde{N}_d} \left[ (\partial_0 - N^l \partial_l) \log \tilde{\psi} - \frac{1}{6} \partial_l N^l \right], \quad (2.46)$$

$$\mathcal{K}[\varphi|e] = \tilde{N}_d \varphi^2 \left( -4 v_{\tilde{\psi}}^2 + \frac{v_{(ab)}^2}{6} \right), \quad (2.47)$$

$$\mathcal{P}[\varphi|e] = \frac{N_d \varphi^2 \tilde{\psi}^7}{6} \left( {}^{(3)}R(\mathbf{e}) \tilde{\psi} + 8 \Delta \tilde{\psi} \right), \quad (2.48)$$

$$\mathcal{S}[\varphi|e] = 2\varphi^2 \left[ \partial_0 v_{\tilde{\psi}} - \partial_l (N^l v_{\tilde{\psi}}) \right] - \frac{\varphi_0^2}{3} \partial_j [\psi^2 \partial^j (\psi^6 N_d)] \quad (2.49)$$

are the kinetic and potential terms, respectively,

$$S_{\text{int}} = -2 \int dx^0 \partial_0 \varphi(x^0) \int d^3x v_{\tilde{\psi}} \quad (2.50)$$

is the interference between the zeroth mode and nonzero ones,

$$S_0 = - \int dx^0 \int d^3x \frac{(\partial_0 \varphi)^2}{\tilde{N}_d} \equiv -V_0 \int dx^0 \frac{(\partial_0 \varphi)^2}{N_0}, \quad (2.51)$$

is the zeroth mode action, and

$$\frac{1}{N_0} \equiv \frac{1}{V_0} \int d^3x \frac{1}{\tilde{N}_d}, \quad (2.52)$$

is the global lapse function.

### 2.1.6 The superfluidity condition

Thus, after the separation of the zeroth mode in the action (2.8) its part describing the spatial metric determinant takes the form

$$S_D = - \int d^4x N_d \left[ 4\varphi^2 (v_{\psi})^2 + 4\varphi v_{\varphi} v_{\psi} + (v_{\varphi})^2 \right], \quad (2.53)$$

where

$$\varphi = \varphi_0 a(x^0) \quad (2.54)$$

$$v_{\varphi} = \partial_0 \varphi / N_d, \quad (2.55)$$

the first term in the Lagrangian arises from the kinetic part Eq. (2.47), the second goes from the “quasi-surface” one (2.49), and the third term goes from the zeroth mode action (2.51). The canonical momentum of the scale factor can be obtained by variation of

Lagrangian (2.53) with respect to the time derivative of scale factor  $\partial_0 \varphi$

$$P_\varphi \equiv \frac{\partial L_{SD}}{\partial(\partial_0 \varphi)} = - \int d^3x [4\varphi v_\psi + 2v_\varphi] \equiv -[4\varphi V_\psi + 2V_\varphi],$$

while the zeroth Fourier harmonics of canonical momentum of the spatial metric determinant is

$$\begin{aligned} P_\psi &\equiv - \int d^3x \frac{\partial \mathcal{L}_{SD}}{\partial(\partial_0 \log \bar{\psi})} = - \int d^3x \bar{p}_\psi = \\ &= - \int d^3x [8\varphi^2 v_\psi + 4\varphi v_\varphi] \equiv -2\varphi[4\varphi V_\psi + 2V_\varphi], \end{aligned} \quad (2.56)$$

where  $V_\varphi = \int d^3x v_\varphi$ ,  $V_\psi = \int d^3x v_\psi$ . These two equations have no solutions as the matrix of the transition from “velocities” to momenta has the zeroth determinant. This means that the “velocities”  $[V_\varphi, V_\psi]$  could not be expressed in terms of the canonical momenta  $[P_\varphi, P_\psi]$  and the Dirac Hamiltonian approach becomes a failure. To be consistent with identity (2.36) and to keep the number of variables of GR, we should impose the strong constraint

$$V_\psi \equiv \int d^3x v_\psi \equiv 0, \quad (2.57)$$

otherwise we shall have the double counting of the zeroth-Fourier harmonics of spatial metric determinant.

A “*double counting*” is replacement of  $L_1 = (\dot{x})^2/2$  by  $L_2 = (\dot{x} + \dot{y})^2/2$ . The second theory is not mathematically equivalent to the first. The test of this nonequivalence is the failure of the Hamiltonian approach to  $L_2 = (\dot{x} + \dot{y})^2/2$ . Therefore, the replacement  $L_1 \rightarrow L_2$  is nonsense in the context of the Hamiltonian approach.

The interference term plays role of friction. If we accept the Landau condition for superfluidity

$$\int d^3x v_\psi = 0, \quad (2.58)$$

then the interference term vanishes. The Landau condition (2.58) is consistent with the Hamiltonian system, because in the opposite case we have the double counting of the zeroth mode component which destroys the Hamiltonian structure of the theory and leads to problems in expressing the velocity by canonical conjugate momentum.

The next example is Lifshitz’s perturbation theory given by Eq. (3.21) p. 217 in [60]

$$ds^2 = a^2(\eta)[(1 + 2\Phi)d\eta^2 - (1 - 2\Psi)\gamma_{ij}dx^i dx^j].$$

This formula contains the double counting of the zeroth Fourier harmonics of the spatial metrics determinant presented by two variables:

the scale factor  $a$  and  $\langle \Psi \rangle = \int d^3x \Psi(\eta, x_i)$  instead of one. Thus, the accepted cosmological perturbation theory is not consistent with the Hamiltonian approach to GR considered above.

N <sup>2</sup>	UNIVERSE	PARTICLE
1.	$S[\varphi_0 F]$ (2.59)	$\tilde{S}_{\text{SR}}[X^0 X^k]$ (2.38)
2.	$d\zeta = dx^0 e_u$	$ds = d\tau e_p$
3.	$x^0 \rightarrow \tilde{x}^0 = \tilde{x}^0(x^0)$	$\tau \rightarrow \tilde{\tau} = \tilde{\tau}(\tau)$
4.	$\varphi(x^0) = \varphi_0 a(x^0)$	$X_0(\tau)$
5.	$\varphi   \tilde{F}$	$X_0   X_k$
6.	$P_\varphi^2 - E_\varphi^2 = 0$	$P_0^2 - E_0^2 = 0$
7.	$\zeta_{(\pm)} = \pm \int_{\varphi_I}^{\varphi_0} d\varphi \langle (T_d)^{-1/2} \rangle \geq 0$	$s_{\pm} = \pm \frac{m}{E} [X_0^0 - X_I^0] \geq 0$
8.	$P_\varphi = \pm 2 \int d^3x (\tilde{T}_d)^{1/2}$	$P_0 = \pm \sqrt{m^2 c^4 +  \vec{p} ^2}$
9.	$[\tilde{P}_\varphi^2 - E_\varphi^2] \Psi_{\text{WDW}} = 0$	$[\tilde{P}_0^2 - E_0^2] \Psi_{\text{KG}} = 0$
10.	$\Psi_{\text{WDW}} = \frac{A^+ + A^-}{\sqrt{2E_\varphi}}$	$\Psi_{\text{KG}} = \frac{a^+ + a^-}{\sqrt{2E_0}}$
11.	$A^+ = \alpha B^+ + \beta^* B^-$	$a^+ = \alpha b^+ + \beta^* b^-$
12.	$B^-  0\rangle = 0$	$b^-  0\rangle = 0$
13.	$\langle 0   A^+ A^-   0 \rangle \neq 0$	$\langle 0   a^+ a^-   0 \rangle \neq 0$

Table 1: **The  $3L + 1G$  diffeomorphisms & universe-particle correspondence** [15, 28]. This universe-particle correspondence rejects Hilbert's *Foundations of relativistic physics* of 1915 [4] that based on the action principle (N<sup>2</sup>1) with a geometric interval (N<sup>2</sup>2) and the group of diffeomorphisms (N<sup>2</sup>3), in contrast to the classical physics based only on an action and the group of the data transformations. The group of diffeomorphisms (N<sup>2</sup>3) leads to the energy constraint (N<sup>2</sup>6). Resolution of the energy constraint gives the Hubble type relation (N<sup>2</sup>7) between the time-variable (N<sup>2</sup>4) in space of events (N<sup>2</sup>5) and the time-interval (N<sup>2</sup>2) and determines the energy of events (N<sup>2</sup>8) that can take positive and negative values. With the aim to remove the negative value, one can use the experience of QFT, i.e., the primary quantization (N<sup>2</sup>9) and the secondary one (N<sup>2</sup>10). This quantization procedure leads immediately to creation from stable Bogoliubov vacuum state (N<sup>2</sup>12) of both quasiuniverses and quasiparticles (N<sup>2</sup>13) obtained by the Bogoliubov transformation (N<sup>2</sup>11) [61, 62].

### 2.1.7 The Hamiltonian formalism in finite space-time

The cosmological perturbation theory is consistent with the Dirac – ADM Hamiltonian approach to GR considered above, if the zeroth time-like variable in the field space of events (2.35) is extracted on

the level of action (2.43), (2.51), (2.52) [15, 28]

$$S[\varphi_0|F] = \tilde{S}[\varphi|\tilde{F}] - V_0 \int dx^0 \frac{1}{N_0} \left( \frac{d\varphi}{dx^0} \right)^2 = \int dx^0 L; \quad (2.59)$$

where  $\tilde{S}[\varphi|\tilde{F}]$  is the action (2.2) in terms of metrics  $\tilde{g}$ , where  $\varphi_0$  is replaced by the running scale  $\varphi(x^0) = \varphi_0 a(x^0)$  of all masses of the matter fields. The action (2.59) leads to the energy constraints

$$\frac{\delta S[\varphi_0]}{\delta \tilde{N}_d} = -T_d = \frac{(\partial_0 \varphi)^2}{\tilde{N}_d^2} - \tilde{T}_d = 0, \quad \tilde{T}_d \equiv -\frac{\delta \tilde{S}[\varphi]}{\delta \tilde{N}_d} \geq 0 \quad (2.60)$$

The kinematic subgroup (2.37) essentially simplifies the solution of the energy constraint (2.60) if the homogeneous variable is extracted from the spacial determinant.

Therefore, one should point out in the finite volume the homogeneous variable  $\varphi(x^0)$  as the evolution parameter (time-variable) in the field space of events  $[\varphi|\tilde{F}]$  and diffeo-invariant time-interval  $N_0 dx^0 = d\zeta$  (g-time), where  $N_0[\tilde{N}_d]$  as functional of  $\tilde{N}_d$  can be defined as the spacial averaging (2.52)

$$\frac{1}{N_0[\tilde{N}_d]} = \frac{1}{V_0} \int \frac{d^3x}{\tilde{N}_d} \equiv \langle \tilde{N}_d^{-1} \rangle. \quad (2.61)$$

According to the Wheeler – DeWitt [11] there is the universe – particle correspondence given in the Table 2.1.6 [15, 28].

This QFT experience illustrates the possibility to solve the problems of the quantum origin of all matter fields in the Early Universe, its evolution, and the present-day energy budget [15, 9, 63]. In order to use this possibility, one should impose a set of requirements on the cosmic motion in the field space of events that follow from the general principles of QFT.

The QFT experience supposes that the action (2.59) can be represented in the *canonical* Hamiltonian form like (1.68)

$$\begin{aligned} S[\varphi_0|F] &= \int dx^0 \left\{ -P_\varphi \partial_0 \varphi + N_0[\tilde{N}_d] \frac{P_\varphi^2}{4V_0} \right\} + \\ &+ \int d^4x \left[ \sum_{\tilde{F}} P_{\tilde{F}} \partial_0 \tilde{F} + \mathcal{C} - \tilde{N}_d \tilde{T}_d \right]. \end{aligned} \quad (2.62)$$

In this case, the energy constraint (2.60) takes the form of the *Friedmann equation*

$$\left[ \frac{d\varphi}{d\zeta} \right]^2 \equiv \varphi'^2 = \left\langle (\tilde{T}_d)^{1/2} \right\rangle^2, \quad (2.63)$$

and the algebraic equation for the diffeo-invariant lapse function

$$\mathcal{N} = \langle (\tilde{N}_d)^{-1} \rangle \tilde{N}_d = \left\langle (\tilde{T}_d)^{1/2} \right\rangle (\tilde{T}_d)^{-1/2}. \quad (2.64)$$

We see that the energy constraint (2.60) removes only one global momentum  $P_\varphi$  in accord to the dimension of the kinematic diffeomorphisms (2.37) that is consistent with the second Nöther theorem.

One can find the evolution of all field variables  $F(\varphi, x^i)$  with respect to  $\varphi$  by variation of the “reduced” action

$$S[\varphi_0]_{P_\varphi = \pm E_\varphi} = \int_{\varphi_I}^{\varphi_0} d\tilde{\varphi} \left\{ \int d^3x \left[ \sum_F P_F \partial_\varphi F \mp 2\sqrt{\tilde{T}_d(\tilde{\varphi})} \right] \right\} \quad (2.65)$$

obtained as the constraint-shell (2.63) values of the Hamiltonian form of the initial action (2.62) [6].

The energy constraints (2.60) and the Hamiltonian reduction (2.65) lead to the definite *canonical rules* of the Universe evolution in the field space of events  $[\varphi|\tilde{F}]$ .

**Rule 1: Causality Principle in the WDW space**  $\frac{d\varphi_I}{d\varphi_0} = 0$  follows from the Hamiltonian reduction (2.65) that gives us the solution of the Cauchy problem and means that initial data  $\varphi_I, \varphi'_I$  do not depend on the Planck value  $\varphi_0, \varphi'_0$ .

**Rule 2: Positive Energy Postulate** follows from the energy constraint (2.60)  $\frac{(\partial_0 \varphi)^2}{\tilde{N}_d^2} = \tilde{T}_d = -\frac{16}{\varphi^2} p_\psi^2 + \dots \geq 0$

$$\tilde{T}_d \geq 0 \rightarrow p_\psi = -\frac{4\varphi^2}{3\tilde{\psi}^6 N_{\text{inv}}} [\partial_j (\tilde{\psi}^6 \mathcal{N}^j) - (\tilde{\psi}^6)'] = 0, \quad (2.66)$$

where  $(\mathcal{N}^j = N^j \langle N_d^{-1} \rangle \neq 0)$ .

**Rule 3: Vacuum Postulate**  $B^-|0\rangle = 0$  restricts the Universe motion in the field space of events

$$\begin{aligned} P_\varphi &\geq 0 \quad \text{for } \varphi_I \leq \varphi_0 \\ P_\varphi &\leq 0 \quad \text{for } \varphi_I \geq \varphi_0. \end{aligned} \quad (2.67)$$

**Rule 4: Lapse Function**  $\mathcal{N} > 0$  follows from the nonzero energy density  $\tilde{T}_d \neq 0$ .

The Rule 1 is not compatible with the Planck epoch [24]

$$\varphi_0 \cdot a_I = \varphi_0 \cdot \frac{a'_0}{\varphi_0} \rightarrow \varphi_I = \frac{\varphi'_0}{\varphi_0} = \mathcal{H}_0 \quad (2.68)$$



in the beginning of the Universe as  $\frac{d\varphi_I}{d\varphi_0} \neq 0$ .

The Rule 2 (2.66) means that the local scalar component  $\tilde{\psi}^2 = \psi^2/a$  has zero momentum and satisfies the equation with Laplacian (instead of the D’alambertian) in accord with the Dirac classification of the radiation-like variables in GR [13]. In other words, the local scalar component cannot be the local dynamic variable as it is proposed for the description of the CMB power spectrum in the acceptable  $\Lambda$ CDM model [25].

The Rule 3 leads to the arrow of the geometric time-interval.

The Rule 4 forbids any zero values of the local lapse function (2.64), so that penetration into a internal region of black hole is not possible because this penetration is accompanied the change of a sign of the local lapse function (2.64) that proposes zero values of the local lapse function.

Let us check the correspondence of the canonical GR with both the QFT in the flat space-time and the classical Newton theory.

## 2.2 Correspondence principle and QFT limits

The correspondence principle [6] as the low-energy expansion of the “*reduced action*” (2.65) over the field density  $T_s$

$$2d\varphi\sqrt{\tilde{T}_d} = 2d\varphi\sqrt{\rho_0(\varphi) + T_s} = d\varphi \left[ 2\sqrt{\rho_0(\varphi)} + \frac{T_s}{\sqrt{\rho_0(\varphi)}} \right] + \dots \quad (2.69)$$

gives the following sum:

$$S^{(+)}|_{\text{constraint}} = S_{\text{cosmic}}^{(+)} + S_{\text{field}}^{(+)} + \dots, \quad (2.70)$$

where

$$S_{\text{cosmic}}^{(+)}[\varphi_I|\varphi_0] = -2V_0 \int_{\varphi_I}^{\varphi_0} d\varphi \sqrt{\rho_0(\varphi)} \quad (2.71)$$

is the reduced cosmological action (2.65), and

$$S_{\text{field}}^{(+)} = \int_{\eta_I}^{\eta_0} d\eta \int_{V_0} d^3x \left[ \sum_F P_F \partial_\eta F - T_s \right] \quad (2.72)$$

is the standard field action in terms of the conformal time:  $d\eta = \frac{d\varphi}{\sqrt{\rho_0(\varphi)}}$ , in the conformal flat space-time with running masses  $m(\eta) = a(\eta)m_0$ .

This expansion shows that the Hamiltonian approach to the General Theory of Relativity in terms of the Lichnerowicz scale-invariant

variables (2.73) identifies the “conformal quantities” with the observable ones including the conformal time  $d\eta$ , instead of  $dt = a(\eta)d\eta$ , the coordinate distance  $r$ , instead of the Friedmann one  $R = a(\eta)r$ , and the conformal temperature  $T_c = Ta(\eta)$ , instead of the standard one  $T$ . Therefore, the scale-invariant variables distinguish the conformal cosmology (CC) [64], instead of the standard cosmology (SC) [24].

### 2.3 Canonical Cosmological Perturbation Theory

In diffeo-invariant formulation of GR in the specific reference frame the scalar potential perturbations can be defined as  $\mathcal{N}^{-1} = 1 + \overline{\nu}$  and  $\tilde{\psi} = e^{\overline{\mu}} = 1 + \overline{\mu} + \dots$ , where  $\overline{\mu}, \overline{\nu}$  are given in the class of functions distinguished by the projection operator  $\overline{F} = F - \langle F \rangle$  ( $\langle \overline{F} \rangle \equiv 0$ ).

The explicit dependence of the metric simplex and the energy tensor  $\tilde{T}_d$  on  $\psi$  can be given in terms of the scale-invariant Lichnerowicz variables [66] introduced in Appendix C (C.2) and

$$\omega_{(0)}^{(L)} = \tilde{\psi}^4 \mathcal{N} d\zeta, \quad \omega_{(b)}^{(L)} = \mathbf{e}_{(b)k} [dx^k + \mathcal{N}^k d\zeta], \quad (2.73)$$

$$\tilde{T}_d = \tilde{\psi}^7 \hat{\Delta} \tilde{\psi} + \sum_I \tilde{\psi}^I a^{\frac{1}{2}-2} \mathcal{T}_I, \quad \mathcal{T}_I \equiv \langle \mathcal{T}_I \rangle + \overline{\mathcal{T}}_I, \quad (2.74)$$

where  $\hat{\Delta} \tilde{\psi} \equiv \frac{4\varphi^2}{3} \partial_{(b)} \partial_{(b)} \tilde{\psi}$  is the Laplace operator and  $\mathcal{T}_I$  is partial energy density marked by the index  $I$  running a set of values  $I = 0$  (stiff), 4 (radiation), 6 (mass), and 8 (curvature) in correspondence with a type of matter field contributions considered in Appendix C (2.24) – (C.23) (except of the  $\Lambda$ -term,  $I = 12$ ). The negative contribution  $-(16/\varphi^2) \overline{p}_\psi^2$  of the spatial determinant momentum in the energy density  $\mathcal{T}_{I=0}$  can be removed by the Dirac constraint [13] of the zeroth velocity of the spatial volume element (2.66)

$$\overline{p}_\psi = -8\varphi^2 \frac{\partial_\zeta \tilde{\psi}^6 - \partial_l [\tilde{\psi}^6 \mathcal{N}^l]}{\tilde{\psi}^6 \mathcal{N}} = 0. \quad (2.75)$$

The diffeo-invariant part of the lapse function  $N_{\text{int}}$  is determined by the local part (2.64) of the energy constraint (2.60) that can be written as

$$\tilde{T}_d = \mathcal{N}^{-2} \rho_{(0)}, \quad \rightarrow \quad \mathcal{N}^{-1} = \sqrt{\tilde{T}_d} \rho_{(0)}^{-1/2}, \quad (2.76)$$

where  $\rho_{(0)} = \left\langle \sqrt{\tilde{T}_d} \right\rangle^2$ . In the class of functions  $\overline{F} = F - \langle F \rangle$ , the classical equation  $\delta S / \delta \log \tilde{\psi} = 0$  takes the form

$$\tilde{\psi} \frac{\delta S}{\delta \tilde{\psi}} = -\tilde{T}_\psi = \tilde{N}_d \tilde{\psi} \frac{\partial \tilde{T}_d}{\partial \tilde{\psi}} + \tilde{\psi} \Delta \left[ \frac{\partial \tilde{T}_d}{\partial \Delta \tilde{\psi}} \tilde{N}_d \right] = 0.$$

Using the property of the deviation projection operator  $\delta S/\delta\bar{\mu} = \bar{D} = D - \langle D \rangle$ , where  $\bar{\mu} = \log \tilde{\psi}$ , we got the following equation

$$7\mathcal{N}\tilde{\psi}^7\hat{\Delta}\tilde{\psi} + \tilde{\psi}\hat{\Delta}[\mathcal{N}\tilde{\psi}^7] + \mathcal{N}\sum_I I\tilde{\psi}^I a^{\frac{I}{2}-2}\mathcal{T}_I = \rho_{(1)}, \quad (2.77)$$

where  $\rho_{(1)} = \left\langle 7\mathcal{N}\tilde{\psi}^7\hat{\Delta}\tilde{\psi} + \tilde{\psi}\hat{\Delta}[\mathcal{N}\tilde{\psi}^7] + \sum_I I\tilde{\psi}^I a^{\frac{I}{2}-2}\mathcal{T}_I \right\rangle$ . Using (2.76) we can write for  $\tilde{\psi}$  a nonlinear equation

$$(\tilde{T}_d)^{-1/2} \left[ 7\tilde{\psi}^7\hat{\Delta}\tilde{\psi} + \sum_I I\tilde{\psi}^I a^{\frac{I}{2}-2}\mathcal{T}_I \right] + \tilde{\psi}\hat{\Delta}[(\tilde{T}_d)^{-1/2}\tilde{\psi}^7] = \rho_{(1)}\rho_{(0)}^{-1/2}.$$

In the infinite volume limit  $\rho_{(n)} = 0$ ,  $a = 1$  Eqs. (2.76) and (2.77) coincide with the equations of the diffeo-variant formulation of GR  $T_d = 0$  and (2.26) considered in Section 2.3.

For the small deviations  $N_{\text{int}}^{-1} = 1 + \bar{\nu}$  and  $\tilde{\psi} = e^{\bar{\mu}} = 1 + \bar{\mu} + \dots$  the first orders of Eqs. (2.76) and (2.77) take the form

$$(-\hat{\Delta} - \rho_{(1)})\bar{\mu} + 2\rho_{(0)}\bar{\nu} = \bar{\mathcal{T}}_{(0)}, \quad (2.78)$$

$$(14\hat{\Delta} + \rho_{(2)})\bar{\mu} - (\hat{\Delta} + \rho_{(1)})\bar{\nu} = -\bar{\mathcal{T}}_{(1)}, \quad (2.79)$$

where

$$\rho_{(n)} = \langle \mathcal{T}_{(n)} \rangle \equiv \sum_I I^n a^{\frac{I}{2}-2} \langle \mathcal{T}_I \rangle \quad (2.80)$$

$$\mathcal{T}_{(n)} = \sum_I I^n a^{\frac{I}{2}-2} \mathcal{T}_I. \quad (2.81)$$

The set of Eqs. (2.94) and (2.94) gives  $\bar{\nu}$  and  $\bar{\mu}$  in the form of a sum

$$\bar{\mu} = \frac{1}{14\beta} \int d^3y [D_{(+)}(x, y)\overline{T_{(+)}(y)} - D_{(-)}(x, y)\overline{T_{(-)}(y)}],$$

$$\bar{\nu} = \frac{1}{2\beta} \int d^3y [(1 + \beta)D_{(+)}(x, y)\overline{T_{(+)}(y)} - (1 - \beta)D_{(-)}(x, y)\overline{T_{(-)}(y)}],$$

where

$$\beta = \sqrt{1 + [\langle \mathcal{T}_{(2)} \rangle - 14\langle \mathcal{T}_{(1)} \rangle]/(98\langle \mathcal{T}_{(0)} \rangle)}, \quad (2.82)$$

$$\overline{T_{(\pm)}} = (7\overline{\mathcal{T}}_{(0)} - \overline{\mathcal{T}}_{(1)}) \pm 7\beta\overline{\mathcal{T}}_{(0)} \quad (2.83)$$

are the local currents,  $D_{(\pm)}(x, y)$  are the Green functions satisfying the equations

$$[\pm\hat{m}_{(\pm)}^2 - \hat{\Delta}]D_{(\pm)}(x, y) = \delta^3(x - y), \quad (2.84)$$

where  $\hat{m}_{(\pm)}^2 = 14(\beta \pm 1)\langle \mathcal{T}_{(0)} \rangle \mp \langle \mathcal{T}_{(1)} \rangle$ .

In the case of point mass distribution in a finite volume  $V_0$  with the zeroth pressure and the density  $\overline{\mathcal{T}}_{(1)} = \frac{\overline{\mathcal{T}}_{(2)}}{6} \equiv \sum_J M_J \left[ \delta^3(x - y_J) - \frac{1}{V_0} \right]$ , solutions (2.82), (2.82) take a very important form

$$\overline{\mu}(x) = \sum_J \frac{r_{gJ}}{4r_J} \left[ \gamma_1 e^{-m_{(+)}(z)r_J} + (1 - \gamma_1) \cos m_{(-)}(z)r_J \right], \quad (2.85)$$

$$\overline{\nu}(x) = \sum_J \frac{2r_{gJ}}{r_J} \left[ (1 - \gamma_2) e^{-m_{(+)}(z)r_J} + \gamma_2 \cos m_{(-)}(z)r_J \right], \quad (2.86)$$

where

$$\gamma_1 = \frac{1 + 7\beta}{14\beta}, \quad \gamma_2 = \frac{(1 - \beta)(7\beta - 1)}{16\beta},$$

$$r_{gJ} = \frac{3M_J}{4\pi\varphi^2}, \quad r_J = |x - y_J|, \quad m_{(\pm)}^2 = \hat{m}_{(\pm)}^2 \frac{3}{4\varphi^2}.$$

The minimal surface (2.66)  $\partial_i[\overline{\psi}^6 \mathcal{N}^i] - (\overline{\psi}^6)' = 0$  gives the shift of the coordinate origin in the process of evolution

$$\mathcal{N}^i = \left( \frac{x^i}{r} \right) \left( \frac{\partial_\zeta V}{\partial_r V} \right), \quad V(\zeta, r) = \int_0^r d\tilde{r} \tilde{r}^2 \tilde{\psi}^6(\zeta, \tilde{r}). \quad (2.87)$$

In the infinite volume limit  $\langle \mathcal{T}_{(n)} \rangle = 0$  these solutions take the standard Newtonian form:  $\overline{\mu} = D \cdot \mathcal{T}_{(0)}$ ,  $\overline{\nu} = D \cdot [14\mathcal{T}_{(0)} - \mathcal{T}_{(1)}]$ ,  $\mathcal{N}^i = 0$  (where  $\hat{\Delta}D(x) = -\delta^3(x)$ ).

## 2.4 Generalization of the Schwarzschild solution

One can see that another choice of variables for scalar potentials rearranges the perturbation series and leads to another result. In order to demonstrate this fact, let us change the lapse function as  $\mathcal{N}\tilde{\psi}^7 = 1 - \overline{\nu}_1$  and keep  $\tilde{\psi} = 1 + \overline{\mu}_1$ . In order to simplify equations of the scalar potentials  $\mathcal{N}, \tilde{\psi}$ , one can introduce new table of symbols:

$$N_s = \psi^7 \mathcal{N}, \quad (2.88)$$

$$T(\tilde{\psi}) = \sum_I \tilde{\psi}^{(I-7)} a^{\frac{I}{2}-2} \mathcal{T}_I, \quad (2.89)$$

$$\rho_{(0)} = \left\langle \sqrt{\tilde{T}_d} \right\rangle^2 = \varphi'^2. \quad (2.90)$$

In terms of these symbols the action (2.43) can be presented as a generating functional of equations of the local scalar potentials  $N_s, \tilde{\psi}$  and field variables  $F$  in terms of diffeo-invariant time  $\zeta$ :

$$S[\varphi_0] = \int d\zeta \int d^3x \left[ \sum_F P_F \partial_\zeta F - N_s \left( \hat{\Delta} \tilde{\psi} + T(\tilde{\psi}) \right) - \frac{\tilde{\psi}^7 \rho_{(0)}}{N_s} \right]. \quad (2.91)$$

The variations of this action with respect to  $N_s, \tilde{\psi}$  lead to equations

$$\hat{\Delta}\tilde{\psi} + T(\tilde{\psi}) = \frac{\tilde{\psi}^7 \rho_{(0)}}{N_s^2}, \quad (2.92)$$

$$\tilde{\psi}\hat{\Delta}N_s + N_s\tilde{\psi}\partial_{\tilde{\psi}}T + 7\frac{\tilde{\psi}^7 \rho_{(0)}}{N_s} = \rho_{(1)}, \quad (2.93)$$

respectively, we have used here the constraint (2.75) and the property of the deviation projection operator  $\delta S/\delta \bar{\mu} = \bar{D} = D - \langle D \rangle$ , according to which  $\rho_{(1)} = \langle \tilde{\psi}\hat{\Delta}N_s + N_s\tilde{\psi}\partial_{\tilde{\psi}}T + 7\tilde{\psi}^7 \rho_{(0)}/N_s \rangle$ .

One can see that in the infinite volume limit  $\rho_{(n)} = \langle \mathcal{T}_I \rangle = 0$  Eqs. (2.92) and (2.93) reduce to the equations of the conventional GR with the Schwarzschild solutions  $\bar{\psi} = 1 + \frac{r_g}{4r}$ ;  $N_s = 1 - \frac{r_g}{4r}$  in empty space, where Eqs. (2.92) and (2.93) become  $\hat{\Delta}\bar{\psi} = 0$ ,  $\hat{\Delta}N_s = 0$ .

For the small deviations  $N_s = 1 - \nu_1$  and  $\tilde{\psi} = 1 + \mu_1$  the first orders of Eqs. (2.92) and (2.93) take the form

$$[-\hat{\Delta} + 14\rho_{(0)} - \rho_{(1)}]\mu_1 + 2\rho_{(0)}\nu_1 = \bar{\mathcal{T}}_{(0)} \\ [7 \cdot 14\rho_{(0)} - 14\rho_{(1)} + \rho_{(2)}]\mu_1 + [-\hat{\Delta} + 14\rho_{(0)} - \rho_{(1)}]\nu_1 = 7\bar{\mathcal{T}}_{(0)} - \bar{\mathcal{T}}_{(1)},$$

where

$$\rho_{(n)} = \langle \mathcal{T}_{(n)} \rangle \equiv \sum_I I^n a^{\frac{I}{2}-2} \langle \mathcal{T}_I \rangle. \quad (2.94)$$

This choice of variables determines  $\bar{\mu}_1$  and  $\bar{\nu}_1$  in the form of a sum

$$\tilde{\psi} = 1 + \frac{1}{2} \int d^3y \left[ D_{(+)}(x, y) \bar{\mathcal{T}}_{(+)}^{(\mu)}(y) + D_{(-)}(x, y) \bar{\mathcal{T}}_{(-)}^{(\mu)}(y) \right], \\ \mathcal{N}\tilde{\psi}^7 = 1 - \frac{1}{2} \int d^3y \left[ D_{(+)}(x, y) \bar{\mathcal{T}}_{(+)}^{(\nu)}(y) + D_{(-)}(x, y) \bar{\mathcal{T}}_{(-)}^{(\nu)}(y) \right],$$

where  $\beta$  are given by Eqs. (2.82)

$$\bar{\mathcal{T}}_{(\pm)}^{(\mu)} = \bar{\mathcal{T}}_{(0)} \mp 7\beta[7\bar{\mathcal{T}}_{(0)} - \bar{\mathcal{T}}_{(1)}], \quad (2.95)$$

$$\bar{\mathcal{T}}_{(\pm)}^{(\nu)} = [7\bar{\mathcal{T}}_{(0)} - \bar{\mathcal{T}}_{(1)}] \pm (14\beta)^{-1} \bar{\mathcal{T}}_{(0)} \quad (2.96)$$

are the local currents,  $D_{(\pm)}(x, y)$  are the Green functions satisfying the equations (2.84) where  $\hat{m}_{(\pm)}^2 = 14(\beta \pm 1)\langle \mathcal{T}_{(0)} \rangle \mp \langle \mathcal{T}_{(1)} \rangle$ . In the finite volume limit these solutions for  $\tilde{\psi}, \mathcal{N}$  coincide with solutions (2.82) and (2.82), where  $\bar{\nu}_1 = \bar{\nu} - 7\bar{\mu}$  and  $\bar{\mu}_1 = \bar{\mu}$ .

In the case of point mass distribution in a finite volume  $V_0$  with the zeroth pressure and the density

$$\bar{\mathcal{T}}_{(0)}(x) = \frac{\bar{\mathcal{T}}_{(1)}(x)}{6} \equiv M \left[ \delta^3(x - y) - \frac{1}{V_0} \right], \quad (2.97)$$

solutions (2.95), (2.95) take a form

$$\tilde{\psi} = 1 + \frac{r_g}{4r} [\gamma_1 e^{-m_{(+)}(z)r} + (1 - \gamma_1) \cos m_{(-)}(z)r], \quad (2.98)$$

$$\mathcal{N}\tilde{\psi}^7 = 1 - \frac{r_g}{4r} [(1 - \gamma_2) e^{-m_{(+)}(z)r} + \gamma_2 \cos m_{(-)}(z)r], \quad (2.99)$$

where  $\gamma_1 = \frac{1+7\beta}{2}$ ,  $\gamma_2 = \frac{14\beta-1}{28\beta}$ ,  $r_g = \frac{3M}{4\pi\varphi^2}$ ,  $r = |x - y|$ . Both choices of variables (2.85), (2.86) and (2.98), (2.99) have spatial oscillations and the nonzero shift of the coordinate origin of the type of (2.87).

In the infinite volume limit  $\langle \mathcal{T}_{(n)} \rangle = 0$ ,  $a = 1$  solutions (2.98) and (2.99) coincide with the isotropic version of the Schwarzschild solutions:  $\tilde{\psi} = 1 + \frac{r_g}{4r}$ ,  $N_{\text{inv}}\tilde{\psi}^7 = 1 - \frac{r_g}{4r}$ ,  $N^k = 0$ . It is of interest to find an exact solution of Eq. (2.78) for different equations of state.

## 2.5 Investigation of CMB fluctuations

### 2.5.1 CMB fluctuation problem

The investigation of CMB fluctuations is one of the highlights of present-day cosmology with far-reaching implications and more precise observations are planned for the near future. Therefore, the detailed investigation of any possible flaw of the standard theory deserves attention and public discussion.

“CMBR anisotropy” in the inflationary model is described in [59, 60, 67] by the decomposition of the metric interval

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \\ &= a^2(\eta) [(1 + 2\Phi) d\eta^2 - 2\mathcal{N}_k dx^k d\eta - (1 - 2\Psi) dx^2 - dx^i dx^j (h_{ij})] \end{aligned} \quad (2.100)$$

associated with the Lifshits cosmological perturbation theory [59]. The comparison of this interval with the exact interval (3.58) gives for scalar components the relations

$$\tilde{\psi} = 1 - \frac{\Psi}{2}, \quad (2.101)$$

$$\mathcal{N}\tilde{\psi}^6 = 1 + \Phi. \quad (2.102)$$

The final expression for the temperature fluctuations induced by scalar fluctuations of the metric components (known as the Sachs-Wolfe (SW) effect) can be written as [68]

$$\begin{aligned} \left( \frac{\Delta T}{T} \right)_s &= \left[ \frac{\delta\rho_r}{4\rho_r} + \Phi + n_i v_b^i \right]_{\eta_i} + \int_{\eta_i}^{\eta_0} d\eta (\Psi' + \Phi') = \\ &= \left[ \frac{\delta\rho_r}{4\rho_r} + (\mathcal{N}\tilde{\psi}^6 - 1) + n_i v_b^i \right]_{\eta_i} + \int_{\eta_i}^{\eta_0} d\eta \left( \frac{\tilde{\psi}'}{2} + (\mathcal{N}\tilde{\psi}^6)' \right). \end{aligned} \quad (2.103)$$

Equation (2.103) is integrated once<sup>2</sup> with respect to  $\eta$  between the time  $\eta_i$  (coinciding with the decoupling time) and the time  $\eta_0$  (coinciding with the present time). Equation (2.103) has three contribution

- the ordinary SW effect given by the first two terms at the right hand side of Eq. (2.103) i.e.  $\delta\rho_r/(4\rho_r)$  and  $\phi$ ;
- the Doppler term (third term in Eq. (2.103));
- the integrated SW effect (last term in Eq. (2.103)).

The ordinary SW effect is due both to the intrinsic temperature inhomogeneities on the last scattering surface and to the inhomogeneities of the metric. On large angular scales the ordinary SW contribution dominates. The Doppler term arises thanks to the relative velocity of the emitter and of the receiver. At large angular scales its contribution is subleading but it becomes important at smaller scales, i.e. in multipole space, for  $\ell \sim 200$  corresponding to the first peak. The induced temperature fluctuations induced by the vector modes of the geometry can be written as

$$\left(\frac{\Delta T}{T}\right)_v = [-\vec{\mathcal{V}} \cdot \vec{n}]_{\mathcal{I}_i}^{\eta_f} + \frac{1}{2} \int_{\eta_i}^{\eta_f} (\partial_i \mathcal{N}_j + \partial_j \mathcal{N}_i) n^i n^j d\eta. \quad (2.104)$$

where  $\mathcal{V}_b^i$  is the rotational component of the baryonic peculiar velocity.

## 2.5.2 Canonical Cosmological Perturbations Theory versus Lifshitz's one

We shall use the definition of the scalar components of the energy momentum tensor (2.74) and (2.77). The energy momentum tensor components are

$$\begin{aligned} \tilde{T}_d &= \frac{4\varphi_0^2 a^2}{3} \tilde{\psi}^7 \Delta \tilde{\psi} + \\ &+ \sum_{I=0,4,6,8,12} a^{I/2-2} \tilde{\psi}^I \mathcal{T}_I = 2(T_{00} - T_{kk}), \end{aligned} \quad (2.105)$$

$$\begin{aligned} \tilde{T}_\psi &= \frac{4\varphi_0^2 a^2}{3} \left\{ 7\mathcal{N} \tilde{\psi}^7 \Delta \tilde{\psi} + \tilde{\psi} \Delta [\mathcal{N} \tilde{\psi}^7] \right\} + \\ &+ \mathcal{N} \sum_{I=0,4,6,8,12} I a^{I/2-2} \tilde{\psi}^I \mathcal{T}_I = 12T_{kk} \end{aligned} \quad (2.106)$$

Let us compare the equations of the canonical perturbation theory (2.90), (2.92) and (2.93) with the Lifshits cosmological perturbation

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<sup>2</sup>Notice that integration by parts is necessary in order to integrate the term  $2\partial_i \phi n^i$ . Recall, in fact that  $d\Phi/d\eta = \Phi' + \partial_i \phi n^i$ .

theory for the scalar components (2.101) and (2.102)

$$\begin{aligned} 4\pi G a^2 T_{00} &= -3\mathcal{H}(\mathcal{H}\Phi + \Psi') + \triangle\Psi \\ 4\pi G a^2 T_{kk} &= 3[(2\mathcal{H}' + \mathcal{H}^2)\Phi + \mathcal{H}\Phi' + \Psi'' + 2\mathcal{H}\Psi'] + \triangle(\Phi - \Psi), \end{aligned} \quad (2.107)$$

here  $\mathcal{H} = a'/a$  in the case of the zeroth vector and tensor components

$$N_k = 0, \quad h_{ij} = 0.$$

One can see that

1. The  $\Lambda$ CDM Model omits the decomposition of the potential energy

$$\sum_{I=0,4,6,8,12} a^{I/2-2} (1 - \Psi/2)^I \mathcal{T}_I, \quad \mathcal{T}_I = \langle \mathcal{T}_I \rangle + \bar{\mathcal{T}}_I \quad (2.108)$$

with respect  $\Psi$  that leads to the effective mass terms in the Hamiltonian linear equations (2.84) (this effective mass is absent in Eq. (2.107)).

2. The  $\Lambda$ CDM Model chooses the gauge  $N_k = 0$  instead of the Dirac minimal surface  $p_\psi = 0, N_k \neq 0$  consistent with the vacuum postulate in the Hamiltonian approach.
3. The action principle of GR in [60] (see the second formula in Eq. (10.7) p. 261) contains the double counting of the zeroth Fourier-harmonics of the spatial metrics determinant presented by two variables:  $a$  and  $\int d^3x \Psi(\eta, x_i) \neq 0$  instead of one. In other words, the  $\Lambda$ CDM Model uses doubling of the zeroth Fourier harmonic of the scalar metric component  $\tilde{\psi} = 1 - \Psi/2$ ,  $\int d^3x \Psi \neq 0$  in the action [60], that destroys the the Hamiltonian approach. Nevertheless, the linear equations (2.107) in the  $\Lambda$ CDM Model satisfy the opposite conditions  $\int d^3x \Psi = 0$  and  $\int d^3x \Phi = 0$ ? This means that the description of the “primordial power spectrum” by the inflationary model is contradictable.

If we impose the constraints

$$\int d^3x p_\psi(\eta, x_i) = 0, \quad \int d^3x \log \tilde{\psi} = 0 \quad (2.109)$$

in order to remove the “double counting”, we shall return back to the Einstein theory, where the equations of  $\Psi$  and  $\Phi$  will not contain the time derivatives that are responsible for the “primordial power spectrum” in the inflationary model.

In the contrast to standard cosmological perturbation theory [59, 60] the diffeo-invariant version of the perturbation theory do not contain time derivatives that are responsible for the CMB “primordial



power spectrum” in the  $\Lambda$ CDM Model [24]. However, the diffeoinvariant version of the Dirac Hamiltonian approach to GR gives another possibility to explain the CMB radiation spectrum and other topical problems of cosmology by cosmological creation of the vector bosons in the Standard Model [15].

### 3 Unification of GR and SM

#### 3.1 The Unification

The action of the SM in the electroweak sector, with presence of the conformally coupled Higgs field can be write in the form

$$S_{\text{SM}} = \int d^4x \sqrt{-g} \left[ \frac{\phi^2}{6} {}^{(4)}R(g) + \mathcal{L}_{\text{Inv}} + \mathcal{L}_{\text{Higgs}} \right], \quad (3.1)$$

that differs from (1.59) by the curvature term.

The acceptable unification of the General Relativity and the Standard Model is considered as the direct algebraical sum of GR (2.2) and SM (1.59) actions

$$S_{\text{GR\&SM}} = S_{\text{GR}} + S_{\text{SM}}. \quad (3.2)$$

in the Riemannian manifold.

#### 3.2 The Newton's law in the GR&SM theory

The General Relativity and the Standard Model reflect almost all physical effects and phenomena revealed by measurements and observations, however, it does not means that the direct sum of the actions of GR an SM lies in agreement with all these effects and phenomena. One can see that the conformal coupling Higgs field  $\phi$  with conformal weight  $n = -1$  distorts the Newton coupling constant in the Hilbert action (2.2)

$$S_{\text{GR+Higgs}} = \int d^4x \sqrt{-g} \left[ - \left( 1 - \frac{\phi^2}{\varphi_0^2} \right) \frac{\varphi_0^2}{6} R(g) + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] \quad (3.3)$$

due to the additional curvature term in the Higgs Lagrangian (3.3)  $1 - \phi^2/\varphi_0^2$ . This distortion changes the Einstein equations and their standard solutions of the Schwarzschild type and other [69, 70, 71].

The coefficient  $1 - \phi^2/\varphi_0^2$  restricts region, where the Higgs field is given, by the condition  $\phi^2 < \varphi_0^2$ , because in other region  $\phi^2 > \varphi_0^2$  the sign before the 4-dimensional curvature is changed in the Hilbert action (2.2).

In order to keep the Einstein theory (2.2), one needs to consider only the field configuration such that  $\phi^2 < \varphi_0^2$ . For this case one can introduce new variables by the Bekenstein–Wagoner transformation [69]

$$g_{\mu\nu} = g_{\mu\nu}^{(\text{B})} \cosh^2 Q \simeq g_{\mu\nu}^{(\text{B})}, \quad (3.4)$$

$$\phi^2 = \varphi_0^2 \sinh^2 Q \simeq \varphi_0^2 Q^2, \quad (3.5)$$

$$s_{(\text{B})} = (\cosh Q)^{-3/2} s \quad (3.6)$$

considered in [70, 71]. These variables restore the initial Einstein–Hilbert action (3.3) with the standard Newton law in the following way

$$S_{\text{GR+Scalar}} = \varphi_0^2 \int d^4x \sqrt{-g_{(\text{B})}} \left[ -\frac{R(g_{(\text{B})})}{6} + g_{(\text{B})}^{\mu\nu} \partial_\mu Q \partial_\nu Q \right]. \quad (3.7)$$

Now it is clear that *the Bekenstein–Wagoner (BW) transformation converts the "conformal coupling" Higgs field with the weight  $n = -1$  into the "minimal coupling" scalar field  $Q$  - an angle of the scalar - scale mixing that looks like a scalar graviton with the conformal weight  $n = 0$ .*

The Planck mass became one more parameter of the Higgs Lagrangian, so that the lowest order of the Lagrangian after the separation of the zeroth Fourier harmonic  $\langle Q \rangle$

$$Q = \langle Q \rangle + \frac{h}{\varphi_0 \sqrt{2}} \quad (3.8)$$

over small  $\langle Q \rangle \ll 1$  reproduces the acceptable Standard Model action (1.76)

$$\begin{aligned} \mathcal{L}_{\text{Higgs}}^\lambda(\langle Q \rangle) &= \varphi_0^2 g_{(\text{B})}^{00} \partial_0 \langle Q \rangle \partial_0 \langle Q \rangle - \langle Q \rangle \varphi_0 \sum_s f_s \bar{s}_{(\text{B})} s_{(\text{B})} + \\ &+ \frac{\langle Q \rangle^2 \varphi_0^2}{4} \sum_v g_v^2 V^2 - 4\lambda \langle Q \rangle^2 \varphi_0^2 h^2. \end{aligned}$$

### 3.3 The GR&SM cosmology

#### 3.3.1 Diffeo-invariant cosmological dynamics

Finally, we got the unified GR&SM theory

$$S_{\text{GR\&SM}} = \int d^4x \sqrt{-g_{(\text{B})}} \left[ -\varphi_0^2 \frac{R(g_{(\text{B})})}{6} + \mathcal{L}_{\text{Inv}}(F) + \mathcal{L}_{\text{Higgs}} \right], \quad (3.9)$$

where  $F = g_{(\text{B})}, W, Z, s$  and Lagrangians are given by Eqs. (1.60), (1.61), (1.62), (1.63), and

$$\mathcal{L}_{\text{Higgs}} = \varphi_0^2 \partial_\mu Q \partial_\nu Q g_{(\text{B})}^{\mu\nu} - \Phi \sum_s f_s \bar{s} s + \frac{\Phi^2}{4} \sum_v g_v^2 V^2, \quad (3.10)$$

where  $\Phi = \Phi(Q) = \varphi_0 \sinh Q$ . This Lagrangian is depend on a one dimensional parameter  $\varphi_0$  only, that is given by Eq. (2.1).

The next step is to clear up the cosmological consequences of the unified theory. The simplest way to made this step is the extraction of the cosmological scale factor  $a(x^0)$  by scale transformations of all

field variables obtained by the WB transformation (3.4), (3.5), and (3.6)

$$g_{(\text{B})} = a^2 \tilde{g}, \quad (3.11)$$

$$W, Z = \tilde{W}, \tilde{Z}, \quad (3.12)$$

$$s_{(\text{B})} = a^{-3/2} \tilde{s}, \quad (3.13)$$

$$\varphi = a\varphi_0. \quad (3.14)$$

In particular, the curvature  $\sqrt{-g_{(\text{B})}} {}^{(4)}R(g_{(\text{B})}) = a^2 \sqrt{-\tilde{g}} {}^{(4)}R(\tilde{g}) - 6a\partial_0 [\partial_0 a \sqrt{-\tilde{g}} \tilde{g}^{00}]$  can be expressed in terms of the new lapse function

$$\tilde{N}_a = [\sqrt{-\tilde{g}} \tilde{g}^{00}]^{-1} \quad (3.15)$$

and spatial metric determinant  $|\tilde{g}^{(3)}|$ . In this case, one can repeat the diffeo-invariant Hamiltonian formulation of the GR presented in the previous Section 2 [28, 29], where  $\log a$  is identified with a zeroth mode as the spatial volume “averaging”

$$\log a = \frac{1}{6V_0} \int d^3x \log |g_{(\text{B})}^{(3)}| \equiv \frac{1}{6} \langle \log |g_{(\text{B})}^{(3)}| \rangle, \quad (3.16)$$

here the finite Lichnerowicz [66] diffeo-invariant volume  $V_0 = \int d^3x$  is introduced<sup>3</sup>. In this case,

$$\log |\tilde{g}^{(3)}| = \log |g_{(\text{B})}^{(3)}| - 6 \log a \quad (3.17)$$

is identified with the nonzero Fourier harmonics that satisfy the constraint

$$\langle \log |\tilde{g}^{(3)}| \rangle \equiv 0. \quad (3.18)$$

A scalar field can be also presented as a sum of a zeroth Fourier harmonics and nonzero ones

$$Q = \langle Q \rangle + \overline{Q}; \quad \langle \overline{Q} \rangle = 0. \quad (3.19)$$

Finally, the action (3.9) takes the form of the sum of nonzero and zeroth-mode-contributions

$$S_{\text{GR\&SM}}[\varphi_0|F, Q] = S_{\text{GR\&SM}}[\varphi|\tilde{F}, \overline{Q}] + S_{\text{zm}}[\varphi|\overline{Q}]; \quad (3.20)$$

here the first action repeats action  $S_{\text{GR\&SM}}[\varphi_0|F, Q]$  (3.9), where  $[\varphi_0|F, Q]$  are replaced by  $[\varphi|\tilde{F}, \overline{Q}]$ , and the second

$$S_{\text{zm}}[\varphi|\overline{Q}] \Big|_{N_0 \neq 1} = \underbrace{V_0 \int dx^0 \frac{1}{N_0} \left[ \varphi^2 \left( \frac{d\langle Q \rangle}{dx^0} \right)^2 - \left( \frac{d\varphi}{dx^0} \right)^2 \right]}_{\text{zeroth-mode contribution}} \equiv \int dx^0 L_{\text{zm}} \quad (3.21)$$

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<sup>3</sup>One should emphasize that modern cosmological models [59] are considered in the finite space and “finite time-interval” in a reference frame identified with the frame of the Cosmic Background Microwave (CMB) radiation.

is the action of the zeroth modes  $\varphi, \langle Q \rangle$ ; here

$$\frac{1}{N_0} = \frac{1}{V_0} \int \frac{d^3x}{\tilde{N}_d} \equiv \left\langle \frac{1}{\tilde{N}_d} \right\rangle \quad (3.22)$$

is the homogeneous component of the lapse function. The action of the local variables in (3.20) determines the correspondent densities for the local variables

$$\begin{aligned} \tilde{T}_d &= -\frac{\delta S_{\text{GR\&SM}}[\varphi_0|\tilde{F}, \overline{Q}]}{\delta \tilde{N}_d}, \\ \tilde{T}_\psi - \langle \tilde{T}_\psi \rangle &= -\tilde{\psi} \frac{\delta S_{\text{GR\&SM}}[\varphi_0|\tilde{F}, \overline{Q}]}{\delta \tilde{\psi}} = 0, \end{aligned} \quad (3.23)$$

where  $\tilde{\psi} = \sqrt[6]{|\tilde{g}^{(3)}|}$  is the Dirac notation of the spatial metric determinant [13].

The action (3.20) coincides with the action of the relativistic mechanics, where the dimension cosmological scale factor plays the role of the external evolution parameter in the field “space of events”  $[\varphi|\tilde{F}, \overline{Q}, \langle Q \rangle]$ , where  $\varphi$  is the time-like variable in this “space”, and  $\tilde{F}, \overline{Q}, \langle Q \rangle$  are the space-like ones.

The action principle for the  $S[\varphi_0|F, Q]$  with respect to the lapse function  $\tilde{N}_d$  gives the energy constraints equation

$$\frac{1}{\mathcal{N}^2(\zeta, x^k)} \left[ \left( \frac{d\varphi(\zeta)}{d\zeta} \right)^2 - \varphi^2(\zeta) \left( \frac{d\langle Q \rangle(\zeta)}{d\zeta} \right)^2 \right] - \tilde{T}_d(\zeta, x^k) = 0, \quad (3.24)$$

where  $\tilde{T}_d$  is given by Eq. (3.23) and

$$\zeta = \int dx^0 N_0 \quad (3.25)$$

is the “diffeo-invariant homogeneous time-interval” with its derivative and

$$\mathcal{N}(\zeta, x^k) = \tilde{N}_d(\zeta, x^k) \langle \tilde{N}_d^{-1} \rangle(\zeta), \quad (3.26)$$

is diffeo-invariant part of the local lapse function with the unit constraint

$$\langle \mathcal{N}^{-1} \rangle \equiv \frac{1}{V_0} \int d^3x \mathcal{N}^{-1} = 1 \quad (3.27)$$

following from the definition of the homogeneous component of the lapse function  $N_0$  given by Eq. (3.22). This equation is the algebraic one with respect to the diffeo-invariant lapse function  $\mathcal{N}$  and has solution satisfying the constraint (3.26)

$$\mathcal{N} = \langle \tilde{T}_d^{1/2} \rangle \tilde{T}_d^{-1/2}. \quad (3.28)$$

The substitution of this solution into the energy constraint (3.24) leads to the cosmological type equation

$$\varphi'^2 = \rho_{\text{tot}}(\varphi) = \rho_{\text{loc}}(\varphi) + \rho_{\text{zm}}(\varphi); \quad (3.29)$$

here the total energy density  $\rho_{\text{tot}}(\varphi)$  is split on the sum of the energy density of local fields (loc) and the zeroth mode (zm) one defined as

$$\rho_{\text{loc}}(\varphi) = \langle (\tilde{T}_{\text{d}})^{1/2} \rangle^2, \quad \rho_{\text{zm}}(\varphi) = \varphi^2 \langle Q \rangle'^2 = \frac{P_{\langle Q \rangle}^2}{4V_0^2 \varphi^2} \quad (3.30)$$

where

$$P_{\langle Q \rangle} = \frac{\partial L_{\text{zm}}}{\partial(\partial_0 \langle Q \rangle)} = 2V_0 \varphi^2 \frac{d\langle Q \rangle}{d\zeta} \equiv 2V_0 \varphi^2 \langle Q \rangle' \quad (3.31)$$

is the scalar field zeroth mode momentum that is an integral of motion of the considered model because the action does not depend on  $\langle Q \rangle$ . The constraint-shell value of the momentum of external time  $\varphi$

$$P_\varphi = \frac{\partial L_{\text{zm}}}{\partial(\partial_0 \varphi)} = 2V_0 \varphi' = \pm 2V_0 \sqrt{\rho_{\text{tot}}(\varphi)} \equiv \mp E_\varphi \quad (3.32)$$

can be considered as the Hamiltonian generator of evolution of all field variables with respect to  $\varphi$  in the “space of events”  $[\varphi | \tilde{F}, \overline{Q}, \langle Q \rangle]$ . The value of the momentum  $P_\varphi = \pm E_\varphi$  onto solutions of the motion equations can be considered as an “energy of the universe”, in accord with the analogy with relativistic mechanics. We can see also that a solution of Eq. (3.32) with respect to diffeo-invariant time-interval  $\zeta$  (3.25)

$$\zeta_\pm = \pm \int_{\varphi_I}^{\varphi} \frac{d\tilde{\varphi}}{\sqrt{\rho_{\text{tot}}(\tilde{\varphi})}} \quad (3.33)$$

is the Hubble law in the exact theory, that includes the initial datum  $\varphi_I = \varphi(\zeta = 0)$ .

### 3.3.2 Zeroth mode sector of GR&SM theory as a “cosmological model”

Let us consider solutions of Eqs. (3.29), (3.30), (3.32), and (3.33) in the case of the zeroth mode sector in the action (3.24), i.e. for  $\rho_{\text{loc}} = 0$ . The zeroth mode sector  $[\varphi | \langle Q \rangle]$  in the action (3.24)

$$S_{\text{zm}} = V_0 \underbrace{\int dx^0 \frac{1}{N_0} \left[ \varphi^2 \left( \frac{d\langle Q \rangle}{dx^0} \right)^2 - \left( \frac{d\varphi}{dx^0} \right)^2 \right]}_{\text{zeroth-mode contribution}} \quad (3.34)$$

is most important at the beginning of the Universe, when all particle like excitations are absent. One can say that at the beginning there were only the “beginning data”  $\varphi_I, \varphi'_I, \langle Q \rangle_I, \langle Q \rangle'_I$ .

$$ds^2 = ds_{\text{WDW}}^2 = a^2(x^0)[(N_0(x^0)dx^0)^2 - (dx^i dx^i)], \quad (3.35)$$

$$a = \varphi/\varphi_0, \quad (3.36)$$

$$\eta = \int dx^0 N_0(x^0). \quad (3.37)$$

The conformal vacuum Higgs effect considered in Section 1, in the cosmological approximation, is described by the action

$$S_{\text{vac}} = V_0 \int dx^0 \left[ \frac{\varphi^2}{N_0} \left( \frac{d\langle Q \rangle}{dx^0} \right)^2 - \frac{1}{N_0} \left( \frac{d\varphi}{dx^0} \right)^2 - N_0 V_{\text{eff}}^{\text{conf}} \right], \quad (3.38)$$

where  $V_{\text{eff}}^{\text{conf}}$  is the Coleman–Weinberg effective potential and is given by the formula (1.79). These action and interval keep the symmetry with respect to reparametrizations of the coordinate evolution parameter  $x^0 \rightarrow \bar{x}^0 = \bar{x}^0(x^0)$ . Therefore, the cosmological model (3.38) can be considered by analogy with a model of a relativistic particle in the Special Relativity (SR) including the Hamiltonian approach to this theory. The canonical conjugate momenta of the theory (3.38) are

$$P_\varphi = 2\varphi'V_0, \quad P_{\langle Q \rangle} = 2\varphi^2 \langle Q \rangle' V_0 \quad (3.39)$$

where  $f' = \frac{df}{d\eta}$ . The Hamiltonian action has a form

$$S_{\text{vac}} = \int dx^0 \left[ P_{\langle Q \rangle} \frac{d\langle Q \rangle}{dx^0} - P_\varphi \frac{d\varphi}{dx^0} - \frac{N_0}{4V_0} (-P_\varphi^2 + E_\varphi^2) \right], \quad (3.40)$$

where

$$E_\varphi = 2V_0 \left[ \frac{P_{\langle Q \rangle}^2}{4V_0^2 \varphi^2} + V_{\text{eff}}^{\text{conf}} \right]^{1/2} \quad (3.41)$$

is treated as the “energy of a universe”.

The classical energy constraint in the model (3.40) is

$$P_\varphi^2 - E_\varphi^2 = 0 \quad (3.42)$$

and repeat completely the cosmological equations in the case of the rigid state equation  $\Omega_{\text{rigid}} = 1$  because due to the unit vacuum-vacuum transition amplitude  $V_{\text{eff}}^{\text{conf}}(\varphi \langle Q_I \rangle) = 0$

$$\varphi_0^2 a'^2 = \frac{P_{Q_I}^2}{4V_0^2 \varphi^2} = H_0^2 \frac{\Omega_{\text{rigid}}}{a^2}, \quad (3.43)$$

where  $P_{Q_I}$  is a constant of the motion

$$P'_{Q_I} = 0, \quad (3.44)$$

because in the equation of motion

$$P'_{\langle Q \rangle} + \frac{d\mathcal{N}_{eff}^{\text{conf}}(\varphi\langle Q \rangle)}{d\langle Q \rangle} = 0, \quad (3.45)$$

the last term is equal zeroth  $\frac{d\mathcal{N}_{eff}^{\text{conf}}(\varphi\langle Q \rangle)}{d\langle Q \rangle} = 0$  if all masses satisfy the Gell-Mann–Oakes–Renner type relation (1.88).

The solution of these equations take the form

$$\varphi(\eta) = \varphi_I \sqrt{1 + 2\mathcal{H}_I \eta}, \quad \langle Q \rangle(\eta) = Q_I + \log \sqrt{1 + 2\mathcal{H}_I \eta}, \quad (3.46)$$

where

$$\varphi_I = \varphi(\eta = 0), \quad \mathcal{H}_I \equiv \frac{\varphi'(\eta = 0)}{\varphi(\eta = 0)} = \frac{P_{\langle Q \rangle}}{2V_0 \varphi_I^2}, \quad (3.47)$$

$$Q_I = \langle Q \rangle(\eta = 0), \quad P_{\langle Q \rangle} = \text{const} \quad (3.48)$$

are the ordinary “free” initial data of the equation of the motion. Besides of the Higgs field  $Q_H$  can be one more (massless) scalar field  $Q_A$  (of the type of axion (A)). In this case

$$\varphi(\eta) = \varphi_I \sqrt{1 + 2\mathcal{H}_I \eta}, \quad (3.49)$$

$$Q_A(\eta) = Q_{AI} + \frac{\mathcal{H}_A}{2\mathcal{H}_I} \log(1 + 2\mathcal{H}_I \eta), \quad (3.50)$$

$$Q_H(\eta) = Q_{HI} + \frac{\mathcal{H}_H}{2\mathcal{H}_I} \log(1 + 2\mathcal{H}_I \eta), \quad (3.51)$$

where  $\varphi_I, \mathcal{H}_I = \sqrt{\mathcal{H}_A^2 + \mathcal{H}_H^2}$  and  $Q_{AI}, \mathcal{H}_A = \frac{P_A}{2V_0 \varphi_I^2}$  and  $Q_{HI}, \mathcal{H}_H = \frac{P_H}{2V_0 \varphi_I^2}$  are free initial data in the CMB frame of reference, in the contrast to the Inflationary model, where  $\varphi_I = \mathcal{H}(\eta = \eta_0)$ . One can see that this main hypothesis of the Inflationary model contradicts to the diffeo-invariant constraint-shell dynamics of the GR, in particular the cosmological model (3.40), where the constraint-shell Hamiltonian action takes a form

$$S_{\text{vac}}^{\text{constraint shell}} = \int_{\varphi_I}^{\varphi_0} d\varphi \left[ P_{\langle Q \rangle} \frac{d\langle Q \rangle}{d\varphi} \mp E_\varphi \right]. \quad (3.52)$$

As it was shown [28, 29, 63, 72] there are initial data of quantum creation of matter at  $z_I + 1 = 10^{15}/3$ , and a value of the Higgs-metric mixing “angle”  $Q_0 \simeq 3 \times 10^{-17}$  is in agreement with the present-day energy budget of the Universe.



### 3.3.3 Quantum universes versus classical ones

The standard pathway from SR to QFT of particles shows us the similar pathway to “QFT” of universes [11, 10] that include the following steps.

1. The Hamiltonian approach: (3.40),
2. Resolution of the energy constraint:  $P_\varphi^2 - E_\varphi^2 = 0$  with respect to  $P_\varphi = \pm E_\varphi$  (3.32),
3. Reduction as substitution of these solutions into action (3.40) gives us the “reduced action”

$$S = \int_{\varphi_I}^{\varphi_0} d\tilde{\varphi} \left[ P_{(Q)} \frac{d(Q)}{d\tilde{\varphi}} \mp E_\varphi \right] \quad (3.53)$$

and the time-interval ( $\eta$ ) – time-variable ( $\varphi$ ) relation (3.33)

$$\eta_\pm = \pm \int_{\varphi_I}^{\varphi} \frac{d\tilde{\varphi}}{\sqrt{\rho_{zm}(\tilde{\varphi})}} = V_0 \left| \frac{\varphi_0^2 - \varphi_I^2}{P_{(Q)}} \right| \quad (3.54)$$

that is treated in cosmology as the Hubble law (and in SR, as the Lorentz transformation),

4. Primary quantization of the energy constraint:  $[\hat{P}_\varphi^2 - E_\varphi^2]\Psi = 0$ , here  $\hat{P}_\varphi = -i\frac{d}{d\varphi}$ ,
5. Secondary quantization of the energy constraint:  $\Psi = \frac{A^+ + A^-}{\sqrt{2E_\varphi}}$ ,
6. The Bogoliubov transformation:  $A^+ = \alpha B^+ + \beta^* B^-$ ,
7. The Bogoliubov vacuum :  $B^-|0\rangle_U = 0$ , and
8. Cosmological creation of  $N_U = {}_U\langle 0|A^+A^-|0\rangle_U$  universes from the Bogoliubov vacuum  $|0\rangle_U$  at  $\eta = 0$  (see in detail Appendices A and B [10]).

The arrow of the time-interval  $\eta \geq 0$  arises at the step 5.) of the decomposition of the wave function onto the sum of the creation operator of a universe going forward  $\varphi \geq \varphi_I$  with positive energy  $P_\varphi \geq 0$ , and the annihilation operator of a universe going backward  $\varphi \leq \varphi_I$  with energy  $P_\varphi \leq 0$ . This “eightfold pathway” shows us that two quantizations 4.) and 5.) are needed, in order to remove the negative energy and provide the stable system [10].

The eight principles are the basis of the fundamental operator quantization as the result of the QFT experience in the twentieth century. The main principle providing this quantization is the “coordinate time reparametrization symmetry of the action” leading to the concepts of energy constraint, space of events, time-event – time-interval relation, particle, quasiparticle, vacuum, and quantum creation from vacuum as a QFT mathematical model of “Big-Bang” considered above. In the model of rigid state, where  $E_\varphi = P_{\langle Q \rangle}/\varphi$ , we have an exact solution for number of created universes (see Eqs. (B.21))

$$N_U = \frac{1}{4P_{\langle Q \rangle}^2 - 1} \sin^2 \left[ \sqrt{P_{\langle Q \rangle}^2 - \frac{1}{4}} \ln \frac{\varphi}{\varphi_I} \right] \neq 0, \quad (3.55)$$

where

$$\varphi = \varphi_I \sqrt{1 + 2H_I \eta} \quad (3.56)$$

and  $\varphi_I, H_I = \varphi'_I/\varphi_I = P_{\langle Q \rangle}/(2V_0\varphi_I^2)$  are the initial data.

There is another version of GR accepted in  $\Lambda$ CDM cosmological perturbation theory [60, 73]. For simplicity, one can compare this version using as example the zeroth mode sector (3.21). Instead of the SR type theory (3.21) in [60, 73] one uses its version obtained by the substitution of the  $N_0 = 1$ ,  $x^0 = \eta$  gauge into the action (3.21)

$$S_{zm}[\varphi|\overline{Q}] \Big|_{N_0=1} = V_0 \int dx^0 \underbrace{\left[ \varphi^2 \left( \frac{d\langle Q \rangle}{dx^0} \right)^2 - \left( \frac{d\varphi}{dx^0} \right)^2 \right]}_{\text{zeroth-mode contribution}} \Big|_{x^0=\eta}. \quad (3.57)$$

In this case, the reparametrization symmetry is postulated on the level of classical equations [74] so that the measurable conformal time becomes an object of reparametrizations  $\eta \rightarrow \tilde{\eta} = \tilde{\eta}(\eta)$  in the contrast to the Dirac definition of measurable quantities as diffeo-invariants. The quantum theory (3.57) does not contain a vacuum as a state with the minimal energy because the corresponding Hamiltonian is not restricted from bottom; therefore, this theory is not stable in contrast to the initial theory (3.21), where the primary and secondary quantizations determine the vacuum as state with the minimal energy of the constraint-shell Universe motion in its space of events.

However, very the problem of unification GR with SM based on the fundamental quantization of relativistic QFT supposes the consideration of SM and GR on equal footing. Therefore, instead of the theory (3.57) without the vacuum postulate accepted in  $\Lambda$ CDM model [60], we shall consider the theory (3.21) with vacuum postulate accepted in SR and GR. The theory (3.21) has a particular quantum solution (3.55), (3.56) predicting an inevitable vacuum creation of a number of “universes” determined by the “free” initial data  $\varphi_I, \mathcal{H}_I$ , and  $\langle Q \rangle_I = Q_0$  including  $P'_{\langle Q \rangle} = 0$  (3.45) at the moment  $\eta = 0$ .

In this case, the corresponding equation of motion in SM admits an arbitrary value of the initial datum  $\langle Q \rangle_I = Q_0$ . These initial data determine the mass spectrum of the SM particles (vector, bosons and fermion) in the SM Lagrangian.

### 3.4 Hamiltonian GR&SM

#### 3.4.1 GR&SM theory in the $3L+1G$ Hamiltonian approach

As we have seen above in Subsection 1.5, that the diffeo-invariant  $3L+1G$  version

$$ds^2 \equiv \omega_{(\alpha)}\omega_{(\alpha)} = a^2\tilde{\omega}_{(\alpha)}\tilde{\omega}_{(\alpha)} = a^2\tilde{\psi}^4 \omega_{(\alpha)}^{(L)}\omega_{(\alpha)}^{(L)}, \quad (3.58)$$

$$\omega_{(0)}^{(L)} = \tilde{\psi}^4 \mathcal{N} d\zeta, \quad (3.59)$$

$$\omega_{(b)}^{(L)} = \mathbf{e}_{(b)i} dx^i + \mathcal{N}_{(b)} d\zeta, \quad (3.60)$$

$$-\frac{\delta S}{\delta N_d} = 0 \Rightarrow \mathcal{N} = \langle \tilde{T}_d^{1/2} \rangle \tilde{T}_d^{-1/2}, \quad (3.61)$$

$$\tilde{T}_d = \frac{4\varphi_0^2 a^2}{3} \tilde{\psi}^7 \Delta \tilde{\psi} + \sum_{I=0,4,6,8,12} a^{I/2-2} \tilde{\psi}^I \mathcal{T}_I, \quad (3.62)$$

$$-\tilde{\psi} \frac{\delta S}{\delta \tilde{\psi}} = 0 \Rightarrow \tilde{T}_\psi - \langle \tilde{T}_\psi \rangle = 0, \quad (3.63)$$

$$\begin{aligned} \tilde{T}_\psi &= \frac{4\varphi_0^2 a^2}{3} \left\{ 7\mathcal{N} \tilde{\psi}^7 \Delta \tilde{\psi} + \tilde{\psi} \Delta [\mathcal{N} \tilde{\psi}^7] \right\} + \\ &+ \mathcal{N} \sum_{I=0,4,6,8,12} I a^{I/2-2} \tilde{\psi}^I \mathcal{T}_I \end{aligned} \quad (3.64)$$

is more adequate to finite volume and finite time of the cosmological dynamics of the Universe as the whole, with the Hubble law (3.33) than the Dirac – ADM  $4L$  version

$$\omega_{(0)} = \psi^6 N_d dx^0, \quad (3.65)$$

$$\omega_{(b)} = \psi^2 \mathbf{e}_{(b)i} (dx^i + N^i dx^0), \quad (3.66)$$

$$-\frac{\delta S}{\delta N_d} \equiv T_d = \frac{4\varphi_0^2}{3} \psi^7 \Delta \psi + \sum_{I=0,4,6,8,12} \psi^I \mathcal{T}_I = 0, \quad (3.67)$$

$$\begin{aligned} -\psi \frac{\delta S}{\delta \psi} &\equiv T_\psi = \frac{4\varphi_0^2}{3} \left\{ 7N_d \psi^7 \Delta \psi + \psi \Delta [N_d \psi^7] \right\} + \\ &+ N_d \sum_{I=0,4,6,8,12} I \psi^I \mathcal{T}_I = 0. \end{aligned} \quad (3.68)$$

Moreover, the diffeo-invariant  $3L+1G$  Hamiltonian approach can be considered as the finite volume generalization of the acceptable Dirac – ADM one with  $4L$  constraints. Both these approaches coincide in

the infinite volume limit  $a = 1, \langle \tilde{T}_d^{1/2} \rangle \rightarrow 0$ . However, the  $4L$  version loses reparametrization time symmetry principle and its direct consequences, such as the evolution parameter  $\varphi_0 a = \varphi$  in the “field space of events” and the “energy of event” that arises in the Hamiltonian constraint-shell action (3.9)

$$S_{\text{GR\&SM}}|_{\text{constraint-shell}} = \int dx^0 \int d^3x \sum_{F=\psi, e, Q} P_F \partial_0 F. \quad (3.69)$$

The kinematic subgroup (2.37) essentially simplifies the solution of the energy constraint (3.24), if the homogeneous variable is extracted from the determinant  $\psi^2(x^0, x^k) = a(\zeta) \tilde{\psi}^2(\zeta, x^k)$  with the additional constraints

$$\int d^3x \log \tilde{\psi} \equiv 0, \quad (3.70)$$

$$\frac{1}{V_0} \int d^3x \frac{1}{\mathcal{N}} \equiv \left\langle \frac{1}{\mathcal{N}} \right\rangle = 1, \quad (3.71)$$

$$|\mathbf{e}_{(b)i}| = 1, \quad \partial_k \mathbf{e}_{(b)}^k = 0, \quad (3.72)$$

$$(\tilde{\psi}^6)' = \partial_{(b)}(\tilde{\psi}^6 \mathcal{N}_{(b)}) \quad (3.73)$$

$$\zeta_{\pm} = \pm \int_{\varphi_I}^{\varphi} \frac{d\tilde{\varphi}}{\sqrt{\langle (\tilde{T}_d)^{1/2} \rangle^2 + P_{\langle Q \rangle}^2 / (2V_0 \varphi_0 a)^2}}. \quad (3.74)$$

The action (3.69) after the separation of the zeroth modes (3.16) and (3.19) takes the form

$$\begin{aligned} (3.69) &= \int dx^0 \left\{ \left[ \int_{V_0} d^3x \sum_{\tilde{F}=\tilde{\psi}, e, \overline{Q}} P_{\tilde{F}} \partial_0 \tilde{F} \right] + P_{\langle Q \rangle} \frac{d\langle Q \rangle}{dx^0} - P_{\varphi} \frac{d\varphi}{dx^0} \right\} \\ &= \int_{\varphi_I}^{\varphi_0} d\varphi \left\{ \left[ \int_{V_0} d^3x \sum_{\tilde{F}=\tilde{\psi}, e, \overline{Q}} P_{\tilde{F}} \partial_{\varphi} \tilde{F} \right] + P_{\langle Q \rangle} \frac{d\langle Q \rangle}{d\varphi} \pm E_{\varphi} \right\} \end{aligned} \quad (3.75)$$

where

$$P_{\varphi} = \pm E_{\varphi} = \pm 2V_0 \sqrt{\langle (\tilde{T}_d)^{1/2} \rangle^2 + P_{\langle Q \rangle}^2 / (2V_0 \varphi_0 a)^2} \quad (3.76)$$

is the constraint-shell Hamiltonian in the “space of events” given by the resolving the energy constraint (3.32).

### 3.5 Correspondence principle

The physical meaning of this constraint-shell Hamiltonian can be revealed in the limit of the tremendous contribution of the homogeneous

energy density

$$\begin{aligned}
2V_0 \langle (\tilde{T}_d)^{1/2} \rangle^2 &= 2V_0 \langle (\rho_s + T_{\text{sm}})^{1/2} \rangle^2 = 2V_0 \rho_s + \int d^3x T_{\text{sm}}, \\
2V_0 \rho_s &\simeq 10^{79} \text{GeV} \gg \int d^3x T_{\text{sm}} \simeq 10^2 \text{GeV}.
\end{aligned} \tag{3.77}$$

In this case the constraint-shell Hamiltonian takes the form

$$\begin{aligned}
P_\varphi &= \pm E_\varphi = \pm 2V_0 \sqrt{\langle (\tilde{T}_d)^{1/2} \rangle^2 + P_{\langle Q \rangle}^2 / (2V_0 \varphi_0 a)^2} = \\
&= \pm \left[ 2V_0 \sqrt{\rho_{\text{cr}}} + \frac{1}{\sqrt{\rho_{\text{cr}}}} \int_{V_0} d^3x T_{\text{sm}} + \dots \right],
\end{aligned} \tag{3.78}$$

$$\rho_{\text{cr}} = \rho_s + \rho_{\text{zm}}, \quad \rho_{\text{zm}} = \frac{P_{\langle Q \rangle}^2}{(2V_0 \varphi_0 a)^2}. \tag{3.79}$$

Using the standard definition of the conformal time in cosmology  $d\varphi = d\eta \sqrt{\rho_{\text{cr}}}$  one can see that the constraint-shell action (3.75)

$$\begin{aligned}
S &= \int_{\varphi_I}^{\varphi_0} d\varphi \left\{ +P_{\langle Q \rangle} \frac{d\langle Q \rangle}{d\varphi} \pm 2V_0 \sqrt{\rho_{\text{cr}}} \right\} + \\
&+ \int_0^{\eta_0} d\eta \left[ \int_{V_0} d^3x \sum_{\tilde{F}=\tilde{\psi}, e, \overline{Q}} P_{\tilde{F}} \partial_\varphi \tilde{F} \pm T_{\text{sm}} \right].
\end{aligned} \tag{3.80}$$

is the sum of action of homogeneous cosmology and the action of the local field theory with the SM Hamiltonian, where all masses are determined by the Higgs-metric “angle”  $\langle Q \rangle$  and the cosmological scale factor  $a(\eta)$ .

The cosmological dynamics in the form of the Hubble law is

$$\eta_\pm = \pm \varphi_0 \int_{\varphi_0 a_I}^{\varphi_0 a} \frac{d\tilde{a}}{\sqrt{\rho_{\text{cr}}(\tilde{a})}}, \tag{3.81}$$

is a one of consequences of the time reparametrization invariance principle. In the homogeneous approximation

$$\begin{aligned}
\rho_{\text{cr}} &= \rho_s + \rho_{\text{zm}} = \\
&= \rho_{0\text{cr}} \sum_{I=0,4,6,8,12} \Omega_I a^{I/2-2}(\eta), \\
\sum_{I=0,4,6,8,12} \Omega_I &= 1,
\end{aligned} \tag{3.82}$$

where  $\Omega_I$  is partial energy density marked by the index  $I$  running a collection of values  $I = 0$  (rigid), 4 (radiation), 6 (mass), 8 (curvature), 12 ( $\Lambda$ -term) in accordance with a type of matter field contributions.

The equation of  $\langle Q \rangle$  take the standard form

$$P'_{\langle Q \rangle} - \frac{dL}{d\langle Q \rangle} = 0. \quad (3.83)$$

In the case of  $\frac{dL}{d\langle Q \rangle} = 0$ ,  $P_{\langle Q \rangle}/(2V_0) = \langle Q \rangle' \varphi_0^2 a^2$  is an integral of motion. Therefore

$$\langle Q \rangle(\eta) = Q_0 + \frac{P_{\langle Q \rangle}}{2\varphi_0^2 V_0} \int_0^\eta \frac{d\tilde{\eta}}{a^2(\tilde{\eta})}. \quad (3.84)$$

The accepted  $\Lambda$ CDM cosmological model arises in the case if  $P_{\langle Q \rangle} \simeq 0$ , when the rigid state is suppressed by the  $\Lambda$ -term  $\Omega_{I=12}$  in the total density (3.82)

$$\Omega_{I=12} a^4(\eta) \gg \frac{\Omega_{I=0}}{a^2(\eta)}. \quad (3.85)$$

At the Planck epoch of the primordial inflation

$$\varphi_I = \varphi_0 a_0 \simeq H_0 \simeq 10^{-61} \varphi_0 \quad (3.86)$$

this means that  $\Lambda$ -term is greater than the rigid, if

$$\Omega_{\Lambda(I=12)} \geq \frac{\Omega_{\text{rigid}(I=0)}}{a_I^6} = 10^{366} \Omega_{\text{rigid}(I=0)}. \quad (3.87)$$

Here a question arises: What is a reason of this strong dominance of the  $\Lambda$ -term, if its contribution is suppressed in  $10^{366}$  times in comparison with the rigid state?

## 4 GR&SM theory as a conformal brane

### 4.1 The Lichnerowicz variables and *relative units* of the dilaton gravitation

One can say that the manifest dependence on the energy density  $T_d$  on the spacial determinant  $\psi$  in the expression (1.68) is equivalent to a choice the Lichnerowicz (L)-coordinates (2.27)  $\omega_{(\mu)}^{(L)}$  and L-variables (2.28), (2.29) as observable ones. The L-observables are physically equivalent with the case when the field with the mass  $m = m_0\psi^2$  is contained in space-time distinguished by the unit spatial metric determinant and the volume element

$$dV^{(L)} = \omega_{(1)}^{(L)} \wedge \omega_{(2)}^{(L)} \wedge \omega_{(3)}^{(L)} = d^3x. \quad (4.1)$$

In terms of the L-variables and L-coordinates  $\varphi_0\psi^2 = w$  the Hilbert action of classical theory of gravitation (2.2) is formally the same as the action of the dilaton gravitation (DG) [75]

$$\begin{aligned} S_{DG}[\hat{g}^w] &= - \int d^4x \frac{\sqrt{-\hat{g}^w}}{6} R(\hat{g}^w) \equiv \\ &\equiv - \int d^4x \left[ \frac{\sqrt{-g}w^2}{6} R(g) - w\partial_\mu(\sqrt{-g}\partial_\nu g^{\mu\nu}) \right], \end{aligned} \quad (4.2)$$

where  $\hat{g}^w = w^2g$  and  $w$  is the dilaton scalar field. This action is invariant with respect to the scale transformations

$$F^{(n)\Omega} = \Omega^n F^{(n)}, \quad g^\Omega = \Omega^2 g, \quad w^\Omega = \Omega^{-1} w. \quad (4.3)$$

One can see that there is a transformation

$$\Omega = \frac{w}{\varphi_0} \quad (4.4)$$

converting the dilaton action (4.6) into the Hilbert one (2.2). In this manner, the CMB frame reveals the possibility to choose the units of measurements in the canonical GR. The dependence on the energy momentum tensors on the spatial determinant *potential*  $\psi$  is completely determined by the Lichnerowicz (L) transformation to the conformal variables (2.27), (2.28), and (2.29). The manifest dependence on the energy density  $T_d$  on the spacial determinant  $\psi$  in the expression (1.68) is equivalent to a choice the L-coordinates (2.27)  $\omega_{(\mu)}^{(L)}$  and L-variables (2.28), (2.29) as observable ones. The L-observables are physically equivalent with the case when the field with the mass  $m = m_0\psi^2$  is contained in space-time distinguished by the unit spatial metric determinant and the volume element (4.1).

In terms of the L-variables and L-coordinates

$$\varphi_0\psi^2 = X_{(0)} \quad (4.5)$$

the Hilbert action of classical theory of gravitation (2.2) is formally the same as the action of the dilaton gravitation (DG) [75]

$$S_{DG} = - \int d^4x \left[ \frac{\sqrt{-g_{(L)}} X_{(0)}^2}{6} R(g_{(L)}) - X_{(0)} \partial_\mu (\sqrt{-g_{(L)}} \partial_\nu g_{(L)}^{\mu\nu} X_{(0)}) \right], \quad (4.6)$$

where  $g_{(L)\mu\nu} = |g^{(3)}|^{-1/3} g_{\mu\nu}$  and  $X_{(0)}$  is the dilaton scalar field. This action supplemented by conformally coupling scalar  $\Phi = X_{(1)}$  field takes the form of a relativistic brane

$$S_{\text{brane}}^{(D=4/N=2)}[X_{(0)}, X_{(1)}] = - \int d^4x \left[ \sqrt{-g} \frac{X_{(0)}^2 - X_{(1)}^2}{6} {}^{(4)}R(g) - X_{(0)} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu X_{(0)}) + X_{(1)} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu X_{(1)}) \right], \quad (4.7)$$

with two external “coordinates” defined as

$$X_{(0)} = \varphi_0 \psi^2, \quad X_{(1)} = \Phi \quad (4.8)$$

in accord with the standard definition of the general action for brane in  $D/N$  dimensions given in [19] by

$$S_{\text{brane}}^{(D/N)} = - \int d^Dx \sum_{A,B=1}^N \eta^{AB} \left[ \sqrt{-g} \frac{X_A X_B}{(D-2)(D-1)} {}^{(D)}R(g) - X_A \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu X_B) \right]. \quad (4.9)$$

In this case, for  $D = 4$ ,  $N = 2$  we have  $\eta^{AB} = \text{diag}\{1, -1\}$ . The hidden conformal invariance of the theory (4.7) admits to replace the Einstein definition of a measurable interval (2.1) in GR by its conformal invariant version as a Weyl-type ratio

$$ds_{(L)}^2 = \frac{ds^2}{ds_{\text{units}}^2}, \quad (4.10)$$

where  $ds_{\text{units}}^2$  is an interval of the units that is defined like the Einstein definition of a measurable interval (2.1) in GR.

This action is invariant with respect to the scale transformations

$$F^{(n)\Omega} = \Omega^n F^{(n)}, \quad g^\Omega = \Omega^2 g, \quad X_{(0)}^\Omega = \Omega^{-1} X_{(0)}. \quad (4.11)$$

One can see that there is a transformation

$$\Omega = X_{(0)} \varphi_0^{-1} \quad (4.12)$$

converting the dilaton action (4.6) into the Hilbert one (2.2). In this manner, the CMB frame reveals the possibility to choose the units of measurements in the canonical GR.



## 4.2 "Coordinates" in brane "superspace of events"

The analogy of the conformally coupling scalar field in GR with a relativistic brane (4.7) allows us to formulate the choice of variables (3.4) and (3.5) as a choice of the “frame” in the brane “superspace of events”

$$\tilde{X}_{(0)} = \sqrt{X_{(0)}^2 - X_{(1)}^2}, \quad (4.13)$$

$$Q = \operatorname{arc} \coth \frac{X_{(0)}}{X_{(1)}} \quad (4.14)$$

As we have seen above the argument in favor of the choice of these variables is the definition of the measurable value of the Newton constant

$$G = \frac{8\pi}{3} \tilde{X}_{(0)}^{-2} \Big|_{\text{present-day}} = \frac{8\pi}{3} \varphi_0^{-2} \quad (4.15)$$

as the present-day value of the “coordinate”  $\tilde{X}_{(0)} = \varphi_0$ .

In the case the action (4.7) takes the form

$$\begin{aligned} S_{\text{brane}}^{(D=4/N=2)}[X_{(0)}, X_{(1)}] = \\ \int d^4x \left[ \sqrt{-g_{(L)}} \tilde{X}_{(0)}^2 \left( -\frac{{}^{(4)}R(g_{(L)})}{6} + g_{(L)}^{\mu\nu} \partial_\mu Q \partial_\nu Q \right) + \right. \\ \left. \tilde{X}_{(0)} \partial_\mu \left( \sqrt{-g_{(L)}} g_{(L)}^{\mu\nu} \partial_\nu \tilde{X}_{(0)} \right) \right]. \end{aligned} \quad (4.16)$$

This form is the brane generalization of the relativistic conformal mechanics

$$S_{\text{particle}}^{(D=1/N=2)}[X_0, Q_0] = \int ds \left[ X_0^2 \left( \frac{dQ_0}{ds} \right)^2 - \left( \frac{dX_0}{ds} \right)^2 \right]; \quad (4.17)$$

$$ds = dx^0 e(x^0) \quad (4.18)$$

considered as a simple example in the Section 3.

The relativistic mechanics (4.17) has two diffeo-invariant measurable times. They are the geometrical interval (4.18) and the time-like variable  $X_0$  in the external “superspace of events”. The relation between these two “times”  $X_0(s)$  are conventionally treated as a relativistic transformation. The main problem is to point out similar two measurable time-like diffeo-invariant quantities in both GR and a brane (4.16).

The brane/GR correspondence (4.7) and special relativity (4.17) allows us to treat an external time as homogeneous component of the time-like external “coordinate”  $\tilde{X}_{(0)}(x^0, x^k)$  identifying this homogeneous component with the cosmological scale factor  $a$  (2.35)

$$\tilde{X}_{(0)}(x^0, x^k) \rightarrow \varphi(x^0) = \varphi_0 a(x^0) \quad (4.19)$$

because this factor is introduced in the cosmological perturbation theory [59] by the scale transformation of the metrics (4.8) too.

The question arises: What is the value of this initial datum  $\varphi_I = \varphi_0 a_I$ , if cosmological factor is treated as one of “degrees of freedom”?

### 4.3 Free initial data versus “Planck’s epoch”

The “degrees of freedom” means that their initial data are beyond equations of motion (i.e. “free” from any theoretical explanation) and they can be defined by only fitting of diffeo-invariant observational data. This “freedom” is main difference of a “theory” from Inflationary Model [24], where these data are determined by fundamental parameters of equations of motion of the type of the Planck mass. In particular, in the Inflationary Model (and  $\Lambda$ CDM Model too), the initial datum  $a_I$  is explained by the constraint

$$a_I = a'_0/\varphi_0 \equiv \mathcal{H}_0/\varphi_0 \quad (4.20)$$

called the Planck epoch. This constraint is justified by the fundamental status of the Planck mass parameter in the initial Einstein – Hilbert action.

However, as we have seen in both the exact GR (3.2), (3.20) and its cosmological approximation (3.21), diffeo-invariant solutions of Einstein equations in GR contain the Planck mass as a multiplier of the cosmological scale factor  $\varphi = \varphi_0 a$ , so that the Planck mass arises in the diffeo-invariant reduced action (3.53) as a present-day datum  $\varphi_0 = \varphi(\eta = \eta_0)$ .

The present-day status of the Planck mass in GR is one of consequences of the diffeo-invariant reduction of theory to the constraint-shell action (3.9).

Therefore, the justification of the Planck epoch, in Inflationary Model, in the form of the constraint (4.20) looks as artefact of the diffeo-non-invariant consideration that is not compatible (as we have seen above) with the practice of SR and the Dirac definition of observables as diffeo-invariants.

Moreover, the Planck’s constraint (4.20) is not compatible with the causality principle of the diffeo-invariant action, in accord to which, in the sum

$$\int_{\varphi_I}^{\varphi_0} d\tilde{\varphi} E_{\tilde{\varphi}} = \int_{\varphi_I}^{\varphi} d\tilde{\varphi} E_{\tilde{\varphi}} + \int_{\varphi}^{\varphi_0} d\tilde{\varphi} E_{\tilde{\varphi}}, \quad (4.21)$$

the initial datum  $\varphi_I$  in the first integral does not depend on the present-day data  $\varphi_0, \varphi'_0 = \mathcal{H}_0 \varphi_0$  in the second integral in contrast with the acceptable treatment (4.20) of the “Planck epoch” as the Early Universe one  $\varphi_I = \mathcal{H}_0$ .

## 5 Observational tests

### 5.1 Test I. The supernova Ia data

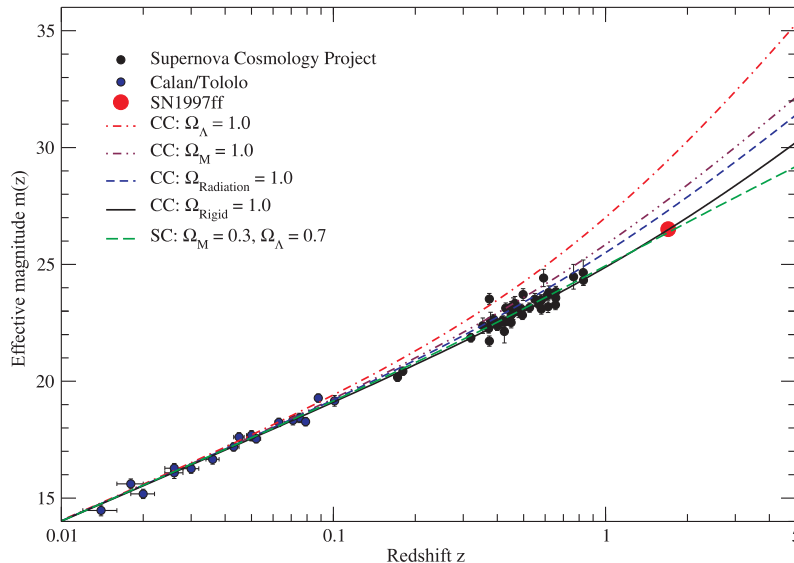


Figure 3: The Hubble diagram in cases of the *absolute* units of standard cosmology (SC) and the *relative* units of conformal cosmology (CC) [65, 76, 77]. The points include 42 high-redshift Type Ia supernovae [86] and the reported farthest supernova SN1997ff [78]. The best fit to these data requires a cosmological constant  $\Omega_\Lambda = 0.7$ ,  $\Omega_{\text{CDM}} = 0.3$  in the case of SC, whereas in CC these data are consistent with the dominance of the rigid (stiff) state. The Hubble Scope Space Telescope team analyzed 186 SNe Ia [79] to test the CC [77].

Since the end of the XX century supernovae data has widespread tested for all theoretical cosmological models. The main reason of this is the fact that supernovae "standard candles" are still unknown or absent [80]. Moreover, the first observational conclusion about accelerating Universe and existence of non-vanishing the  $\Lambda$ -term was done with the cosmological SNe Ia data. Therefore, typically standard (and alternative) cosmological approaches are checked with the test.

Models of Conformal Cosmology are also discussed among other possibilities [8, 10, 15, 28, 29, 63, 64, 76, 81]. Conformal Cosmology is an alternative description of the Supernovae data without the  $\Lambda$ -term as evidence for the Weyl geometry [26] with the relative units interval  $ds_{\text{Weyl}}^2 = ds_{\text{Einstein}}^2 / ds_{\text{Einstein Units}}^2$  where all measurable quantities and their units are considered on equal footing. There is the scalar

version of the Weyl geometry described by the conformal-invariant action of a massless scalar field [75] with the negative sign that is mathematically equivalent to the Hilbert action of the General Relativity where the role of the scalar field  $\phi$  is played by the parameter of the scale transformation  $g^\Omega = \Omega^2 g$  multiplied by the Planck mass  $\phi = \Omega M_{\text{Planck}} \sqrt{3/(8\pi)}$  [82].

We have seen above that the correspondence principle [6] as the low-energy expansion of the “*reduced action*” (2.65) over the field density shows that the Hamiltonian approach to the General Theory of Relativity in terms of the Lichnerowicz scale-invariant variables (2.73) identifies the “conformal quantities” with the observable ones including the conformal time  $d\eta$ , instead of  $dt = a(\eta)d\eta$ , the coordinate distance  $r$ , instead of the Friedmann one  $R = a(\eta)r$ , and the conformal temperature  $T_c = Ta(\eta)$ , instead of the standard one  $T$ . Therefore, the scale-invariant variables distinguish the conformal cosmology (CC) [64, 85], instead of the standard cosmology (SC). In this case, the red shift of the wave lengths of the photons emitted at the time  $\eta_0 - r$  by atoms on a cosmic object in the comparison with the Earth ones emitted at the time  $\eta_0$ , where  $r$  is the distance between the Earth and the object:

$$\frac{\lambda_0}{\lambda_{\text{cosmic}}(\eta_0 - r)} = \frac{a(\eta_0 - r)}{a(\eta_0)} \equiv a(\eta_0 - r) = \frac{1}{1 + z}. \quad (4.22)$$

This red shift can be explained by the running masses  $m = a(\eta)m_0$  in action (2.72). In this case, the Schrödinger wave equation

$$\left[ \frac{\hat{p}_r^2}{2a(\eta)m_0} - \frac{\alpha}{r} \right] \Psi_L(\eta, r) = \frac{d}{i d\eta} \Psi_L(\eta, r) \quad (4.23)$$

can be converted by the substitution  $r = \frac{R}{a(\eta)}$ ,  $p_r = P_R a(\eta)$ ,  $a(\eta)d\eta = dt$ ,  $a(\eta)\Psi_L(\eta, r) = \Psi_0(t, R)$  into the standard Schrödinger wave equation with the constant mass

$$\left[ \frac{\hat{P}_R^2}{2m_0} - \frac{\alpha}{R} \right] \Psi_0(t, R) = \frac{d}{i dt} \Psi_0(t, R). \quad (4.24)$$

Returning back to the Lichnerowicz variables  $\eta, r$  we obtain the spectral decomposition of the wave function of an atom with the running mass

$$\Psi_L(\eta, r) = \frac{1}{a(\eta)} \sum_{k=1}^{\infty} e^{-i\varepsilon_0^{(k)} \int_{\eta}^{\eta_0} d\tilde{\eta} a(\tilde{\eta})} \Psi_0^{(k)}(a(\eta)r) = \sum_{k=1}^{\infty} \Psi_L^{(k)}(\eta, r) \quad (4.25)$$

Where  $\varepsilon_0^{(k)} = \alpha^2 m_0 / k^2$  is a set of eigenvalues of the Schrödinger wave equation in the Coulomb potential. We got the equidistant spectrum

$-i(d/d\eta)\Psi_L^{(k)}(\eta, r) = \varepsilon_0^{(k)}\Psi_L^{(k)}(\eta, r)$  for any wave lengths of cosmic photons remembering the size of the atom at the moment of their emission.

The conformal observable distance  $r$  loses the factor  $a$ , in comparison with the nonconformal one  $R = ar$ . Therefore, in the case of CC, the redshift – coordinate-distance relation  $\eta = \int d\varphi(\sqrt{\rho_0(\varphi)})^{-1}$  corresponds to a different equation of state than in the case of SC [64]. The best fit to the data, including Type Ia supernovae [86, 78], requires a cosmological constant  $\Omega_\Lambda = 0.7$ ,  $\Omega_{\text{CDM}} = 0.3$  in the case of the “scale-variant quantities” of standard cosmology. In the case of “conformal quantities” in CC, the Supernova data [86, 78] are consistent with the dominance of the stiff (rigid) state,  $\Omega_{\text{Rigid}} \simeq 0.85 \pm 0.15$ ,  $\Omega_{\text{Matter}} = 0.15 \pm 0.15$  [64, 65, 76]. If  $\Omega_{\text{Rigid}} = 1$ , we have the square root dependence of the scale factor on conformal time  $a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)}$ . Just this time dependence of the scale factor on the measurable time (here – conformal one) is used for description of the primordial nucleosynthesis [76, 106].

This stiff state is formed by a free scalar field when  $E_\varphi = 2V_0\sqrt{\rho_0} = \frac{Q}{\varphi}$ . In this case there is an exact solution of the Bogoliubov equations of the number of universes created from a vacuum with the initial data  $\varphi(\eta = 0) = \varphi_I, H(\eta = 0) = H_I$  [9].

## 5.2 Test II. Particle creation and the present-day energy budget

These initial data  $\varphi_I$  and  $H_I$  are determined by the parameters of matter cosmologically created from the Bogoliubov vacuum at the beginning of a universe  $\eta \simeq 0$ . The Standard Model (SM) density  $T_s$  in action (2.72) shows us that W-, Z- vector bosons have maximal probability of this cosmological creation due to their mass singularity [63]. One can introduce the notion of *a particle in a universe* if the Compton length of a particle defined by its inverse mass  $M_I^{-1} = (a_I M_W)^{-1}$  is less than the universe horizon defined by the inverse Hubble parameter  $H_I^{-1} = a_I^2(H_0)^{-1}$  in the stiff state. Equating these quantities  $M_I = H_I$  one can estimate the initial data of the scale factor  $a_I^2 = (H_0/M_W)^{2/3} = 10^{-29}$  and the primordial Hubble parameter  $H_I = 10^{29}H_0 \sim 1 \text{ mm}^{-1} \sim 3K$ . Just at this moment there is an effect of intensive cosmological creation of the vector bosons described in paper [63] (see Fig. 2); in particular, the distribution functions of the longitudinal vector bosons demonstrate clearly a large contribution of relativistic momenta. Their conformal (i.e. observable) temperature  $T_c$  (appearing as a consequence of collision and scattering of these bosons) can be estimated from the equation in the kinetic theory for the time of establishment of this temperature  $\eta_{\text{relaxation}}^{-1} \sim n(T_c) \times \sigma \sim H$ , where  $n(T_c) \sim T_c^3$  and  $\sigma \sim 1/M^2$  is the

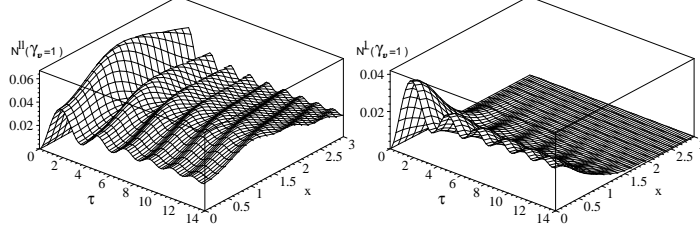


Figure 4: Dependence of longitudinal  $N^{\parallel}$  and transverse  $N^{\perp}$  components of the distribution function of vector bosons on time  $\tau = 2h_i\eta$  and momentum ( $x = q/m_i$ ). Their momentum distributions in units of the primordial mass  $x = q/M_I$  show the large contribution of longitudinal bosons. Values of the initial data  $M_I = H_I$  follow from the uncertainty principle and give the temperature of relativistic bosons  $T \sim H_I = (M_0^2 H_0)^{1/3} = 2.7K$  [63]

cross-section. This kinetic equation and values of the initial data  $M_I = H_I$  give the temperature of relativistic bosons

$$T_c \sim (M_I^2 H_I)^{1/3} = (M_0^2 H_0)^{1/3} \sim 3K \quad (4.26)$$

as a conserved number of cosmic evolution compatible with the Supernova data [64, 78, 86]. We can see that this value is surprisingly close to the observed temperature of the CMB radiation  $T_c = T_{\text{CMB}} = 2.73 \text{ K}$ .

The primordial mesons before their decays polarize the Dirac fermion vacuum (*as the origin of axial anomaly* [87, 88, 89, 90]) and give the baryon asymmetry frozen by the CP – violation. The value of the baryon–antibaryon asymmetry of the universe following from this axial anomaly was estimated in paper [63] in terms of the coupling constant of the superweak-interaction

$$n_b/n_\gamma \sim X_{CP} = 10^{-9}. \quad (4.27)$$

The boson life-times  $\tau_W = 2H_I\eta_W \simeq (2/\alpha_W)^{2/3} \simeq 16$ ,  $\tau_Z \sim 2^{2/3}\tau_W \sim 25$  determine the present-day visible baryon density

$$\Omega_b \sim \alpha_W = \frac{\alpha_{QED}}{\sin^2 \theta_W} \sim 0.03. \quad (4.28)$$

All these results (4.26) – (4.28) testify to that all visible matter can be a product of decays of primordial bosons, and the observational data on CMB radiation can reflect parameters of the primordial bosons, but not the matter at the time of *recombination*. In particular, the length of the semi-circle on the surface of the last emission of photons

at the life-time of W-bosons in terms of the length of an emitter (i.e.  $M_W^{-1}(\eta_L) = (\alpha_W/2)^{1/3}(T_c)^{-1}$ ) is  $\pi \cdot 2/\alpha_W$ . It is close to  $l_{min} \sim 210$  of CMB radiation, whereas  $(\Delta T/T)$  is proportional to the inverse number of emitters  $(\alpha_W)^3 \sim 10^{-5}$ .

The temperature history of the expanding universe copied in the “conformal quantities” looks like the history of evolution of masses of elementary particles in the cold universe with the constant conformal temperature  $T_c = a(\eta)T = 2.73$  K of the Cosmic Microwave Background radiation.

Equations of the vector bosons in SM are very close to the equations of the  $\Lambda$ CDM model with the inflationary scenario used for description of the CMB “power primordial spectrum”.

### 5.3 Test III: The Newton potential and the Large-scale structure

The equations describing the longitudinal vector bosons in SM, in this case, are close to the equations that follow from the Lifshits perturbation theory and are used, in the inflationary model, for description of the “power primordial spectrum” of the CMB radiation.

The next differences are a nonzero shift vector and spatial oscillations of the scalar potentials determined by  $\hat{m}_{(-)}^2$ . In the conformal cosmology model [64], the SN data corresponds to the dominance of rigid state  $\Omega_{rigid} \sim 1$ . The rigid state determines the parameter of spatial oscillations

$$\hat{m}_{(-)}^2 = \frac{6}{7}H_0^2[\Omega_R(z+1)^2 + \frac{9}{2}\Omega_{Mass}(z+1)]. \quad (4.29)$$

The redshifts in the recombination epoch  $z_r \sim 1100$  and the clustering parameter [82]

$$r_{clustering} = \frac{\pi}{\hat{m}_{(-)}} \sim \frac{\pi}{H_0\Omega_R^{1/2}(1+z_r)} \sim 130 \text{ Mpc} \quad (4.30)$$

recently discovered in the researches of a large scale periodicity in redshift distribution [83, 84] lead to a reasonable value of the radiation-type density  $10^{-4} < \Omega_R \sim 3 \cdot 10^{-3} < 5 \cdot 10^{-2}$  at the time of this epoch.

## 6 Summary

We have proposed

- to make the Bekenstein – Wagoner transformation in the unified GR&SM theory, in order to restore the Newton law, (this restoration converts the Higgs field into dimensionless "angle" of the metric-Higgs mixing),
- to convert the fundamental constant in the Higgs potential onto the zeroth Fourier harmonic of the Higgs field, that predicts mass of the Higgs field by initial date of the zeroth Fourier harmonic,
- to attach the cosmological scale factor to the Planck mass as the single dimension parameter of the theory,
- to choose the Conformal Cosmology description [8, 10, 15, 28, 29, 63, 64, 76, 77, 81] where the Rigid state is an equivalent of the Quintessence state in the Standard Cosmology and gives satisfactory explanation of the last Supernovae Ia data for the luminosity distance-redshift relation without a cosmological constant,
- to choose the free initial data for cosmological scale factor instead of the Early Universe Planck epoch where initial data of the scale factor is determined by its present-day velocity (*i.e.* the Hubble parameter). In the class of the diffeo-invariant solutions of the Einstein field equations, the Planck epoch can be unambiguously treated as the present-day one, whereas the opposite case contradicts to the Causality Principle.

In the Section 4 it was shown that there are free initial data of the Electro-Weak epoch for both the zeroth modes (scale factor and "angle" of the metric-Higgs mixing) that initiated the intensive vacuum creation of the SM particles that can be treated as the "Big-Bang".

In this case the Hilbert action-interval principle with two times—the time of events as the cosmological scale factor, and the time-interval gives responses on almost all problems of the Inflationary Model, if we replace its Early Universe Planck epoch by the free initial data of the Electro-Weak epoch.

- The Hubble law is the time of events – time-interval relation;
- Energy of events is the scale factor canonical momentum;
- Homogeneity is consequence of the zeroth mode metric excitation obtained by averaging of the spacial metric determinant logarithm over the volume;



- Horizon is given by the Weyl definition of the measurable interval that attaches the cosmological scale factor to all masses so that we get running masses instead of the expanded Universe;
- Flatness is given by the initial data;
- Planck era is the present-day one;
- Singularity is absent in Quantum Universe with stable vacuum;
- Arrow of time-interval is consequence of the causal quantization and primary and secondary quantization of the energy constraint, in order to obtain the QFT Bogoliubov vacuum, as a state with the minimal energy of events;
- Cosmological vacuum creation treated in the modern literature as "Big-Bang" is consequence of the running masses;
- Origins of temperature ( 2.7 K) are consequence of scattering and collisions of primordial particle created from vacuum.

The unified theory keeps the Newton gravity, provided that the Higgs scalar field mixes with the scalar metric component that convert the Higgs field into the "metric-Higgs mixing angle"  $Q$ .

Then, in order to adapt the Standard Model (SM) to cosmology, we consider the SM version, where the fundamental dimensional parameter  $C$  in the Higgs potential  $\lambda(\Phi^2 - C^2)^2$  is replaced by zeroth Fourier harmonic of the Higgs field  $\Phi$ , so that all masses in SM are determined by initial data of the potential free (i.e. *inertial*) equation of this harmonic. Such the replacement of the fundamental parameter by an initial datum immediately predicts mass of Higgs field  $\sim 250$  GeV that follows from the extremum of the quantum Coleman – Weinberg effective potential obtained from the unit vacuum–vacuum transition amplitude in the *inertial* motion regime. We consider a cosmological model, where the difference between the Higgs field and any additional scalar field forming the density of the Early Universe can be explained by initial data of *inertial* motions of these scalar fields. The inertial motion of a scalar field corresponds to the rigid state equation. The dominance of the rigid state is consistent with the best fit to the Hubble diagram of high-redshift Type Ia supernovae and SN1997ff [79] in the Conformal Cosmology [8, 10, 15, 28, 29, 63, 64, 76, 77, 81], whereas in Standard Cosmology these data requires a cosmological constant  $\Omega_\Lambda = 0.7$ ,  $\Omega_{\text{ColdDarkMatter}} = 0.3$  [25].

The Conformal Cosmology is the generalization of the Copernican relativity of positions and velocities to the Weyl relativity of values of intervals. The Conformal Cosmology is based on the Weyl definition of the measurable interval as the ratio of the Einstein one and the

units determined by the standard mass  $m_0$ . If we joint the cosmological scale factor to the length, we get the expanded Universe in the Standard Cosmology. If we joint the cosmological scale factor to the units (i.e. masses including  $m_0$ ), we get the Conformal Cosmology as the *collapsed units of an observer*, just as Copernicus explained the Planet motion by a *relative position of an observer* in the Heliocentric system. In both the cases, the *Copernican relativity of positions and velocities* and the *Weyl relativity of values* of intervals mean free initial data of equations of motion, in particular, the free initial data of the cosmological scale factor in contrast to the Planck epoch of the Inflationary Model [24]. All masses in Conformal Cosmology depend on the initial data of a solution of the equation of motions in the CMB frame, like a trajectory of a particle in the Newton mechanics depends on initial data fitted from observations in a “frame of reference to initial data”.

One more difference from the Inflationary Model [24] is the *Einstein – Hilbert relativity of times*. The *relativity of times* allows us to treat cosmological scale factor as the time-variable in space of events, cosmological equations – as the energy constraint in space of events, and Big-Bang as an inevitable consequence of the primary and secondary quantization of this energy constraint in the form of vacuum creation of the universe and matter with the arrow of time-interval as an quantum anomaly. This anomaly is the consequence of the vacuum postulate about a quantum state with the minimal energy in the space of events.

All relativity principles are consistent with the Dirac Hamiltonian approach based on the classification of all field components onto the Laplace-like *potentials* with the boundary condition and the d’Alambert-like *degrees of freedom* with initial data.

The CMB frame fixing and the finite space-time suppose a formulation of GR with the complete separation of the frame transformations from the general coordinate ones identified with the diffeomorphisms [9, 28]. Such the separation states the following questions.

1. What is a status of the conformal time in the Hubble redshift – luminosity distance relation? Is the conformal time an object of Bardeen’s gauge transformations [74], or it is gauge-invariant measurable quantity in CMB frame in accord with the Dirac definition of observable quantities as invariants?
2. What is a status of the cosmological scale factor? Is it an additional variable in Lifshitz’s cosmological perturbation theory [60], or it is a zeroth mode of the scalar metric component?
3. What are initial data of the cosmological scale factor? Are they the Planck epoch data as the origin of the numerous problems in the Inflationary Model [24], or they are free from equations

of motion in accord with the standard causality principle for any dynamic systems?

4. What is the “vacuum” of cosmological models?
5. What is the origin of arrow of time?

Responses to these questions can be given by the mentioned above the Hamiltonian tool [13, 28, 29, 91].

Just the complete separation of the frame transformations from the general coordinate ones is the main difference of the Hamiltonian approach to GR from the naive Bardeen’s approach [74] to the cosmological perturbation theory accepted in the Inflationary Model. The Bardeen’s gauge transformations of the measurable time  $\eta \rightarrow \tilde{\eta} = \tilde{\eta}(\eta)$  identifies the unmeasurable coordinate time  $x^0$  as an object of the reparametrizations with the diffeo-invariant measurable interval  $\eta = x^0$ . This confusing  $x^0 = \eta$  is an obstacle for the consistent description of the dynamics in the reduce phase space of events  $[\varphi|Q]$ , where the scale factor  $\varphi$  is the single independent “evolution parameter”. This confusing two times  $x^0 = \eta$  also prevents to understand the relativistic status of the Hubble law, space of events  $[\varphi|Q]$ , the energy of events, the vacuum of events, the vacuum creation of the Universe, and the arrow of time in the cosmological models [10].

We show that there are the free initial data  $a_I = 3 \times 10^{-15}$ ,  $Q_0 = 3 \times 10^{-17}$  that give the similar inevitable vacuum creation of particle in agreement with the present day energy budget of the Universe.

## Acknowledgements

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## A Hilbert's QFT Foundations

### A.1 Hilbert's formulation of Special Relativity

The Hilbert geometric formulation of a relativistic particle [4, 6, 7] is based on the action:

$$S_{1915}^{\text{SR}} = -\frac{m}{2} \int_{\tau_1}^{\tau_2} d\tau \left[ \frac{(\dot{X}_\alpha)^2}{e(\tau)} + e(\tau) \right], \quad (\text{A.1})$$

and an *geometric interval*

$$ds = e(\tau)d\tau \quad \longmapsto \quad s(\tau) = \int_0^\tau d\bar{\tau} e(\bar{\tau}), \quad (\text{A.2})$$

where  $\tau$  is the *coordinate evolution parameter* given in a one-dimensional Riemannian manifold with a single component of the metrics  $e(\tau)$  and the variables  $X_\alpha$  form the Minkowskian *space of events*, where  $(X_\alpha)^2 = X_0^2 - X_i^2$ .

The action (A.1) and interval (A.2) are invariant with respect to reparametrizations of the *coordinate evolution parameter*  $\tau \rightarrow \tilde{\tau} = \tilde{\tau}(\tau)$ ; therefore, the theory given by (A.1) and (A.2) can be considered as the simplest model of GR. A single component of the metrics  $e(\tau)$  plays the role of the Lagrange multiplier in the Hamiltonian form of the action (A.1):

$$S_{1915}^{\text{SR}} = \int_{\tau_1}^{\tau_2} d\tau \left[ -P_\alpha \partial_\tau X^\alpha + \frac{e(\tau)}{2m} (P_\alpha^2 - m^2) \right]. \quad (\text{A.3})$$

Varying action (A.3) over lapse-function  $e(\tau)$  defines the “*constraint*”:

$$(P_\alpha)^2 - m^2 = 0. \quad (\text{A.4})$$

Varying action (A.3) over dynamical variables  $(P_\alpha, X_\alpha)$  gives the equations of motion:  $P_\alpha = m dX_\alpha/ds$ ,  $dP_\alpha/ds = 0$ , taking into consideration  $ds = e(\tau)d\tau$ . Solutions of these equations in terms of gauge-invariant geometric interval (A.2) take the form

$$X_\alpha(s) = X_\alpha(0) + \frac{P_\alpha(0)}{m} s. \quad (\text{A.5})$$

The physical meaning of this solution is revealed in a specific “*frame of reference*”. In particular, solutions of energy constraint (A.4) with respect to a temporal component  $P_0$  of momentum  $P_\alpha$

$$P_{0\pm} = \pm \sqrt{m^2 + P_i^2} \quad (\text{A.6})$$

are considered as the “*reduced Hamiltonian*” in the “*reduced phase space*”  $\{X_i, P_j\}$  that becomes the energy  $E(P) = \sqrt{m^2 + P_i^2}$  onto a trajectory [2, 3]. The time component of solution (A.5)

$$s = \frac{m}{P_{0\pm}}[X_0(s) - X_0(0)] \quad (\text{A.7})$$

shows us that the “*time-like variable*”  $X_0$  is identified with the time measured in the rest frame of reference, whereas an interval  $s$  is the time measured in the comoving frame.

The dynamic version of SR [2, 3] can be obtained as values of the geometric action (A.3) onto solutions of the constraint (A.6)

$$S_{1915}^{\text{SR}}|_{P_0=P_{0\pm}} = S_{1905}^{\text{SR}} = \int_{X_{0I}}^{X_0} d\bar{X}_0 \left[ P_i \frac{dX_i}{d\bar{X}_0} - P_{0\pm} \right]. \quad (\text{A.8})$$

Just the values of the “*geometric interval*” (A.7) and action (A.8) onto resolutions (A.6) of constraint (A.4) in the specific frame of reference will be called the “*Hamiltonian reduction*” of Hilbert’s geometric formulation of SR given by Eqs. (A.1) and (A.2) (see [6, 13]).

## A.2 Dynamic Special Relativity of 1905

The “*Hamiltonian reduction*” leads to action (A.8) of the dynamic theory of a relativistic particle of “1905” [2, 3] that establishes a correspondence with the classical mechanic action by the low-energy decomposition

$$E(P) = \sqrt{m^2 + P_i^2} = m + \frac{P_i^2}{2m} + \dots \quad (\text{A.9})$$

It gives us the very important concept of particle “energy”  $E(0) = mc^2$ . We can see that relativistic relation (A.7) between the “time as the variable” and the “time as the interval” appears in the geometric version of “1915” [4] as a consequence of the variational equations (A.6), whereas in the dynamic version of “1905” [2, 3] the same relativistic relation in the form of a kinematic Lorenz relativistic transformation is supplemented to variational equations following from the dynamic action (A.8).

## A.3 Quantum geometry of a relativistic particle

The next step forward to QFT is the primary quantization of particle variables:  $i[\hat{P}_\mu, X_\nu] = \delta_{\mu\nu}$ , that leads to the quantum version of the energy constraint (A.4)  $[\square + m^2]\psi(X_0, X_i) = 0$  known as the Klein – Gordon equation of the wave function. The general solution of this equation

$$\partial_0^2 \Psi_p + E_p^2 \Psi_p = 0 \quad (\text{A.10})$$

for a single p-Fourier harmonics  $\Psi_p(X_0) = \int d^3X \exp i P_j X^j \psi(X_0, X_i)$  takes the form of the sum of two terms

$$\Psi_p = \frac{1}{\sqrt{2E_p}} \{a_p^{(+)}(X_0) + a_p^{(-)}(X_0)\},$$

where  $a_p^{(+)}(X_0), a_p^{(-)}(X_0)$  are solutions of the equations

$$(i\partial_0 + E_p)a_p^{(+)} = 0, \quad (i\partial_0 - E_p)a_p^{(-)} = 0. \quad (\text{A.11})$$

They are treated as the Shrödinger equations of the dynamic theory (A.8) for the case of positive and negative particle “energies” (A.6) revealed by resolving energy constraint (A.4).

QFT is formulated as the secondary quantization of a relativistic particle  $[a_p^{(-)}, a_p^{(+)}] = 1$  [62]. In order to remove the negative “energy”  $-E_p$  and to provide the quantum system with stability, the field  $a_p^{(+)}$  is considered as the operator of creation of a particle and  $a_p^{(-)}$  as the operator of annihilation of a particle, both with positive “energy”. The initial datum  $X_{I(0)}$  is treated as a point of this creation or annihilation. This interpretation means postulating vacuum as a state with minimal “energy”  $a_p^{(-)}|0\rangle = 0$ , and it restricts the motion of a particle in the space of events, so that a particle with  $P_{0+}$  moves forward and with  $P_{0-}$  backward.

$$P_{0+} \rightarrow X_{I(0)} \leq X_{(0)}; \quad P_{0-} \rightarrow X_{I(0)} \geq X_{(0)}. \quad (\text{A.12})$$

As a result of such a restriction the interval (A.7) becomes

$$s_{(P_{0+})} = \frac{m}{E_p} [X_0(s) - X_0(0)]; \quad X_{I(0)} \leq X_{(0)}, \quad (\text{A.13})$$

$$s_{(P_{0-})} = \frac{m}{E_p} [X_0(0) - X_0(s)]; \quad X_{I(0)} \geq X_{(0)}. \quad (\text{A.14})$$

One can see that in both cases the geometric interval is positive. In other words, the stability of quantum theory and the vacuum postulate as its consequence lead to the absolute reference point of this interval  $s = 0$  and its positive arrow. The last means violation of the symmetry of classical theory with respect to the transformation  $s \rightarrow -s$ . Recall that the violation of the symmetry of classical theory by their quantization is called the quantum anomaly [87, 88, 89]. The quantum anomaly as the consequence of the vacuum postulate was firstly discovered by Jordan [90] and then rediscovered by a lot of authors (see [87]).

#### A.4 Creation of particles

Creation of particles is described by QFT obtained by quantization of classical fields with masses depending on time  $m = m(X_0)$ . Classical

field equation (A.10) can be got by varying the action

$$S_p = \int dX_0 \{P_p \partial_0 \Psi_p - H_p\}, \quad (\text{A.15})$$

where  $H_p = \frac{1}{2} [P_p^2 + E_p^2(X_0) \Psi_p^2]$  is the field Hamiltonian, here we kept only one p-harmonics.

The holomorphic representation of the fields [94, 95]

$$\Psi_p = \frac{1}{\sqrt{2E_p(X_0)}} \{a_p^{(+)}(X_0) + a_p^{(-)}(X_0)\}, \quad (\text{A.16})$$

$$P_p = i\sqrt{\frac{E_p(X_0)}{2}} \{a_p^{(+)}(X_0) - a_p^{(-)}(X_0)\}. \quad (\text{A.17})$$

allows us to express the field Hamiltonian in action (A.15) in terms of observable quantities — the one-particle energy  $E_p(X_0)$  and “number” of particles  $N_p(X_0) = [a_p^+ a_p^-]$ :

$$H_p = \frac{1}{2} [P_p^2 + E_p^2(X_0) \Psi_p^2] = E(X_0) \left[ N_p(X_0) + \frac{1}{2} \right]. \quad (\text{A.18})$$

While the canonical structure  $P_p \partial_0 \Psi$  in (A.15) takes the form:

$$P_p \partial_0 \Psi_p = \left[ \frac{i}{2} (a_p^+ \partial_0 a_p^- - a_p^- \partial_0 a_p^+) - \frac{i}{2} (a_p^+ a_p^+ - a_p^- a_p^-) \frac{\partial_0 E(X_0)}{2E(X_0)} \right].$$

The one-particle energy and the number of particle are not conserved. In order to find a set of integrals of motion, we can use the Bogoliubov transformations

$$a_p^+ = \alpha b_p^+ + \beta^* b_p^-, \quad (\alpha = e^{i\theta} \cosh r, \quad \beta = e^{-i\theta} \sinh r), \quad (\text{A.19})$$

so that the equations of  $b_p^+, b_p^-$  become diagonal

$$(i\partial_0 + E_B) b_p^+ = 0, \quad (i\partial_0 - E_B) b_p^- = 0, \quad (\text{A.20})$$

and the conserved vacuum is defined by the postulate:

$$b_p^- |0\rangle_p = 0. \quad (\text{A.21})$$

The corresponding Bogoliubov equations of diagonalization expressed in terms of the distribution function of “particles”  $N_p(X_0)$  and the rotation function  $R_p(X_0)$

$$\begin{aligned} N_p(X_0) &= |\beta|^2 \equiv {}_p \langle a_p^+ a_p^- \rangle_p \equiv \sinh^2 r, \\ R_p(X_0) &= i(\alpha \beta^* - \alpha^* \beta) \equiv -\sin(2\theta) \sinh 2r \end{aligned}$$

take the form [94]

$$\begin{cases} \frac{dN_p}{dX_0} = \frac{\partial_0 E(X_0)}{2E(X_0)} \sqrt{4N_p(N_p + 1) - R_p^2}, \\ \frac{dR_p}{dX_0} = -2E(X_0) \sqrt{4N_p(N_p + 1) - R_p^2}, \end{cases} \quad (\text{A.22})$$

$$E_B(X_0) = \frac{E_p(X_0) - \partial_0 \theta}{\cosh 2r}. \quad (\text{A.23})$$

These equations supplemented by the quantum geometric interval (A.13) and (A.14) are the complete set of equations for description of the phenomenon of particle creation.

Thus, the direct way from Hilbert's geometric formulation of any relativistic theory to the corresponding "quantum field theory" goes through Dirac's Hamiltonian reduction and Bogoliubov's transformations. As a result, we have the description of creation of a relativistic particle in the space of events at the absolute reference point of geometric interval  $s$  of this particle. The physical meaning of this interval is revealed in the Quantum Cosmology considered below.

## B Quantum universes

### B.1 QFT of universes

After the primary quantization of the cosmological scale factor  $\varphi$ :  $i[P_\varphi, \varphi] = 1$  the energy constraint  $P_\varphi^2 = E_\varphi^2$  transforms to the WDW equation

$$\partial_\varphi^2 \Psi + E_\varphi^2 \Psi = 0. \quad (\text{B.1})$$

This equation can be obtained in the corresponding classical WDW field theory for universes of the type of the Klein – Gordon one:

$$S_U = \int d\varphi \frac{1}{2} [(\partial_\varphi \Psi)^2 - E_\varphi^2 \Psi^2] \equiv \int d\varphi L_U. \quad (\text{B.2})$$

Introducing the canonical momentum  $P_\Psi = \partial L_U / \partial(\partial_\varphi \Psi)$ , one can obtain the Hamiltonian form of this theory

$$S_U = \int d\varphi \{P_\Psi \partial_\varphi \Psi - H_U\}, \quad (\text{B.3})$$

where

$$H_U = \frac{1}{2} [P_\Psi^2 + E_\varphi^2 \Psi^2]. \quad (\text{B.4})$$

is the Hamiltonian. The concept of the one-universe "energy"  $E_\varphi$  gives us the opportunity to present this Hamiltonian  $H_U$  in the standard forms of the product of this "energy"  $E_\varphi$  and the "number" of universes

$$N_U = A^+ A^-, \quad (\text{B.5})$$



$$H_U = E_\varphi \frac{1}{2} [A^+ A^- + A^- A^+] = E_\varphi [N_U + \frac{1}{2}] \quad (\text{B.6})$$

by means of the transition to the holomorphic variables

$$\Psi = \frac{1}{\sqrt{2E_\varphi}} \{A^+ + A^-\}, \quad P_\Psi = i\sqrt{\frac{E_\varphi}{2}} \{A^+ - A^-\}. \quad (\text{B.7})$$

The dependence of  $E_\varphi$  on  $\varphi$  leads to the additional term in the action expressed in terms the holomorphic variables

$$P_\Psi \partial_\varphi \Psi = \frac{i}{2} (A^+ \partial_\varphi A^- - A^- \partial_\varphi A^+) - \frac{i}{2} (A^+ A^+ - A^- A^-) \Delta(\varphi), \quad (\text{B.8})$$

where

$$\Delta(\varphi) = \frac{\partial_\varphi E_\varphi}{2E_\varphi}. \quad (\text{B.9})$$

The last term in (B.8) is responsible for the cosmological creation of “universes” from “vacuum”.

## B.2 Bogoliubov transformation. Creation of universes

In order to define stationary physical states, including a “vacuum”, and a set of integrals of motion, one usually uses the Bogoliubov transformations [18, 94, 95] of the variables of universes ( $A^+, A^-$ ):

$$A^+ = \alpha B^+ + \beta^* B^-, \quad A^- = \alpha^* B^- + \beta B^+ \quad (|\alpha|^2 - |\beta|^2 = 1), \quad (\text{B.10})$$

so that the classical equations of the field theory in terms of universes

$$(i\partial_\varphi + E_\varphi)A^+ = iA^- \Delta(\varphi), \quad (i\partial_\varphi - E_\varphi)A^- = iA^+ \Delta(\varphi), \quad (\text{B.11})$$

take a diagonal form in terms of *quasiuniverses*  $B^+, B^-$ :

$$(i\partial_\varphi + E_B(\varphi))B^+ = 0, \quad (i\partial_\varphi - E_B(\varphi))B^- = 0. \quad (\text{B.12})$$

The diagonal form is possible, if the Bogoliubov coefficients  $\alpha, \beta$  in Eqs. (B.10) satisfy to equations

$$(i\partial_\varphi + E_\varphi)\alpha = i\beta\Delta(\varphi), \quad (i\partial_\varphi - E_\varphi)\beta^* = i\alpha^*\Delta(\varphi). \quad (\text{B.13})$$

For the parametrization

$$\alpha = e^{i\theta(\varphi)} \cosh r(\varphi), \quad \beta^* = e^{i\theta(\varphi)} \sinh r(\varphi), \quad (\text{B.14})$$

where  $r(\varphi), \theta(\varphi)$  are the parameters of “squeezing” and “rotation”, respectively, Eqs. (B.13) become

$$\begin{aligned} (i\partial_\varphi \theta - E_\varphi) \sinh 2r(\varphi) &= -\Delta(\varphi) \cosh 2r(\varphi) \sin 2\theta(\varphi), \\ \partial_\varphi r(\varphi) &= \Delta(\varphi) \cos 2\theta(\varphi), \end{aligned} \quad (\text{B.15})$$

while “energy” of *quasiuniverses* in Eqs. (B.12) is defined by expression

$$E_B(\varphi) = \frac{E_\varphi - \partial_\varphi \theta(\varphi)}{\cosh 2r(\varphi)}. \quad (\text{B.16})$$

Due to Eqs. (B.12) the “number” of *quasiuniverses*  $\mathcal{N}_B = B^+ B^-$  is conserved

$$\frac{d\mathcal{N}_B}{d\varphi} \equiv \frac{d(B^+ B^-)}{d\varphi} = 0. \quad (\text{B.17})$$

Therefore, we can introduce the “vacuum” as a state without *quasiuniverses*:

$$B^- |0\rangle_U = 0. \quad (\text{B.18})$$

A number of created *universes* from this Bogoliubov vacuum is equal to the expectation value of the operator of the *number of universes* (B.5) over the Bogoliubov vacuum

$$N_U(\varphi) = {}_U \langle A^+ A^- \rangle_U \equiv |\beta|^2 = \sinh^2 r(\varphi), \quad (\text{B.19})$$

where  $\beta$  is the coefficient in the Bogoliubov transformation (B.10), and  $N_U(\varphi)$  is called the “distribution function”. Introducing the Bogoliubov “condensate”

$$R_U(\varphi) = i(\alpha\beta^* - \alpha^*\beta) \equiv {}_U \langle P_\Psi \Psi \rangle_U = \frac{i}{2} {}_U \langle [A^+ A^+ - A^- A^-] \rangle_U, \quad (\text{B.20})$$

one can rewrite the Bogoliubov equations of the diagonalization (B.13)

$$\begin{cases} \frac{dN_U}{d\varphi} = \Delta(\varphi) \sqrt{4N_U(N_U + 1) - R_U^2}, \\ \frac{dR_U}{d\varphi} = -2E_\varphi \sqrt{4N_U(N_U + 1) - R_U^2}. \end{cases} \quad (\text{B.21})$$

It is natural to propose that at the moment of creation of the universe  $\varphi(\eta = 0) = \varphi_I$  both these functions are equal to zeroth  $N_U(\varphi = \varphi_I) = R_U(\varphi = \varphi_I) = 0$ . This moment of the conformal time  $\eta = 0$  is distinguished by the vacuum postulate (B.18) as the beginning of a universe.

### B.3 Quantum anomaly of conformal time

As it was shown in the case of a particle in QFT [62], the postulate of a vacuum as a state with minimal “energy” restricts the motion of a “universe” in the space of events, so that a “universe” with  $P_{\varphi+}$  moves forward and with  $P_{\varphi-}$  backward.

$$P_{\varphi+} \rightarrow \varphi_I \leq \varphi_0; \quad P_{\varphi-} \rightarrow \varphi_I \geq \varphi_0. \quad (\text{B.22})$$

If we substitute this restriction into the interval (3.37)

$$\eta_{(P_{vh+})} = \int_{\varphi_I}^{\varphi_0} \frac{d\varphi}{\sqrt{\rho_0(\varphi)}}; \quad \varphi_I \leq \varphi_0, \quad (\text{B.23})$$

$$\eta_{(P_{\varphi-})} = \int_{\varphi_0}^{\varphi_I} \frac{d\varphi}{\sqrt{\rho_0(\varphi)}}; \quad \varphi_I \geq \varphi_0, \quad (\text{B.24})$$

one can see that the geometric interval in both cases is positive. In other words, the stability of quantum theory as the vacuum postulate leads to the absolute point of reference of this interval  $\eta = 0$  and its positive arrow. In QFT the initial datum  $\varphi_I$  is considered as a point of creation or annihilation of universe. One can propose that the singular point  $\varphi = 0$  belongs to antiuniverse. In this case, a universe with a positive energy goes out of the singular point  $\varphi = 0$ .

In the model of rigid state  $\rho = p$ , where  $E_\varphi = Q/\varphi$  Eqs. (B.21) have an exact solution

$$N_U = \frac{1}{4Q^2 - 1} \sin^2 \left[ \sqrt{Q^2 - \frac{1}{4}} \ln \frac{\varphi}{\varphi_I} \right] \neq 0, \quad (\text{B.25})$$

where

$$\varphi = \varphi_I \sqrt{1 + 2H_I \eta} \quad (\text{B.26})$$

and  $\varphi_I, H_I = \varphi'_I/\varphi_I = Q/(2V_0\varphi_I^2)$  are the initial data.

We see that there are results of the type of the arrow of time and absence of the cosmological singularity (B.23), which can be understood only on the level of quantum theory, where symmetry  $\eta \rightarrow -\eta$  is broken [87, 88].

## C Massive electrodynamics in GR

As the model of the matter let us consider massive electrodynamics in GR

$$S = \int d^4x \sqrt{-g} \left[ -\frac{\varphi_0^2}{6} R(g) + \mathcal{L}_m \right] \equiv \int d^4x \sqrt{-g} \mathcal{L}, \quad (\text{C.1})$$

where  $\mathcal{L}_m$  is the Lagrangian of the massive vector and spinor fields

$$\mathcal{L}_m = -\frac{1}{4} F_{\mu\nu} F_{\alpha\beta} g^{\mu\alpha} g^{\nu\beta} - M_0^2 A_\mu A_\nu g^{\mu\nu} - \tilde{\psi} i \gamma^\sigma (D_\sigma - ie A_\sigma) \Psi - m_0 \tilde{\psi} \hat{\Psi}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the stress tensor,  $D_\delta = \partial_\delta - i\frac{1}{2}[\gamma_{(\alpha}\gamma_{\beta)}]\sigma_{\delta(\alpha)(\beta)}$ , is the Fock covariant derivative[12],  $\gamma_{(\beta)} = \gamma^\mu e_{(\beta)\mu}$  are the Dirac  $\gamma$ -matrices, summed with tetrads  $e_{(\beta)\nu}$ , and  $\sigma_{\sigma(\alpha)(\beta)} = e_{(\beta)}^\nu (\nabla_\mu e_{(\alpha)\nu})$  are coefficients of spin-connection [12, 96].

The Lagrangian of the massive fields (C.2) can be rewritten in terms of the Lichnerowicz variables

$$A^L_{\mu} = A_{\mu}, \quad \Psi^L = a^{3/2} \psi^3 \Psi, \quad (\text{C.2})$$

that lead to fields with masses depending on the scale factor  $a\psi^2$

$$m_{(\text{L})} = m_0 a \psi^2 = m \psi^2, \quad M_{(\text{L})} = M_0 a \psi^2 = M \psi^2. \quad (\text{C.3})$$

These fields are in the space defined by the component of the frame

$$\omega_{(0)}^{(\text{L})} = \tilde{\psi}^4 \tilde{N}_d dx^0, \quad (\text{C.4})$$

$$\omega_{(a)}^{(\text{L})} = \mathbf{e}_{(a)i} (dx^i + N^i dx^0). \quad (\text{C.5})$$

with the unit metric determinant  $|\mathbf{e}| = 1$ .

As the result, the Lagrangian of the matter fields (C.2) takes the form

$$\begin{aligned} \sqrt{-g} \mathcal{L}_m(A, \tilde{\psi}, \Psi) &= \frac{1}{i} \tilde{\psi}^L \gamma^0 \left( \partial_0 - N^k \partial_k + \frac{1}{2} \partial_l N^l - ie A_0 \right) \Psi^L \\ &- \tilde{N}_d \left[ \tilde{\psi}^6 m \tilde{\psi}^L \Psi^L + \mathcal{H}_{\Psi} \right] - \left[ \tilde{N}_d \frac{\pi_0^2}{M^2} + \tilde{\psi}^8 M^2 A_{(b)}^2 \right] - \pi_0 [N^i A_i - A_0] \\ &+ \tilde{N}_d \left[ -J_{5(c)} v_{[ab]} \varepsilon_{(c)(a)(b)} + \frac{\tilde{\psi}^4}{2} \left( v_{i(\text{A})} v_{(\text{A})}^i - \frac{1}{2} F_{ij} F^{ij} \right) \right], \end{aligned}$$

where the Legendre transformation  $A_0^2/(2\tilde{N}_d) = \pi_0 A_0 - \tilde{N}_d \pi_0^2/2$  with the subsidiary field  $\pi_0$  is used for linearizing the massive term;

$$\mathcal{H}_{\Psi} = -\tilde{\psi}^4 [i \tilde{\psi}^L \gamma_{(b)} D_{(b)} \Psi^L + J_5^0 \sigma - \partial_k J^k] \quad (\text{C.6})$$

is the Hamiltonian density of the fermions,

$$v_{[ab]} = \frac{1}{2} \left( \mathbf{e}_{(a)i} v_{(b)}^i - \mathbf{e}_{(b)i} v_{(a)}^i \right), \quad (\text{C.7})$$

$$D_{(b)} \Psi^L = [\partial_{(b)} - \frac{1}{2} \partial_k \mathbf{e}_{(b)}^k - ie A_{(b)}] \Psi^L, \quad (\text{C.8})$$

$$v_{i(\text{A})} = \frac{1}{\psi^4 \tilde{N}_d} [\partial_0 A_i - \partial_i A_0 + F_{ij} N^j] \quad (\text{C.9})$$

are the field velocities, and

$$J_{5(c)} = \frac{i}{2} (\tilde{\psi}^L \gamma_5 \gamma_{(c)} \Psi^L), \quad (\text{C.10})$$

$$J_5^0 = \frac{i}{2} (\bar{\Psi}^L \gamma_5 \gamma^0 \Psi^L), \quad (\text{C.11})$$

$$J_k = \frac{i}{2} \bar{\Psi}^L \gamma_k \Psi^L \quad (\text{C.12})$$

are the currents,  $\sigma = \sigma_{(a)(b)|(c)}\varepsilon_{(a)(b)(c)}$ , where  $\varepsilon_{(a)(b)(c)}$  denotes the Levi-Civita tensor.

The canonical conjugated momenta take the form

$$P_\varphi = -2V_0 \frac{\partial_0 \varphi}{N_0} = -2V_0 \frac{d\varphi}{d\zeta} \equiv -2V_0 \varphi' \quad (\text{C.13})$$

$$\overline{p_\psi} = \frac{\partial[\sqrt{-g}\mathcal{L}]}{\partial(\partial_0 \ln \tilde{\psi})} = -8\varphi^2 \tilde{v}, \quad (\text{C.14})$$

$$p_{(b)}^i = \frac{\partial[\sqrt{-g}\mathcal{L}]}{\partial(\partial_0 \mathbf{e}_{(a)i})} = \mathbf{e}_{(a)}^i \left[ \frac{\varphi^2}{3} v_{(ab)} - J_{5(c)} \varepsilon_{(c)(a)(b)} \right], \quad (\text{C.15})$$

$$P_{(A)}^i = \frac{\partial[\sqrt{-g}\mathcal{L}]}{\partial(\partial_0 A_i)} = \tilde{\psi}^4 v_{(A)}^i, \quad (\text{C.16})$$

$$P_{(\Psi)} = \frac{\partial[\sqrt{-g}\mathcal{L}]}{\partial(\partial_0 \Psi^L)} = \frac{1}{i} \tilde{\psi}^L \gamma^0. \quad (\text{C.17})$$

Then, the action (C.1) one can be represented in the Hamiltonian form

$$S = \int dx^0 \left[ -P_\varphi \partial_0 \varphi + N_0 \frac{P_\varphi^2}{4V_0} + \int d^3x \left( \sum_F P_F \partial_0 F + \mathcal{C} - \tilde{N}_d T_{0t}^0 \right) \right],$$

where  $P_F$  is a set of the field momenta (C.14) – (C.17),

$$T_{0t}^0 = \tilde{\psi}^7 \hat{\Delta} \tilde{\psi} + \sum_{I=0,4,6,8} \tilde{\psi}^I \tau_I, \quad (\text{C.18})$$

is the sum of the Hamiltonian densities including the gravity density

$$\tilde{\psi}^7 \hat{\Delta} \tilde{\psi} \equiv \tilde{\psi}^7 \frac{4\varphi^2}{3} \partial_{(b)} \partial_{(b)} \tilde{\psi}, \quad (\text{C.19})$$

$$\tau_{I=0} = \frac{6\tilde{p}_{(ab)}\tilde{p}_{(ab)}}{\varphi^2} - \frac{16}{\varphi^2} \overline{p_\psi}^2 + \frac{\pi_0^2}{2a^2 M^2}, \quad (\text{C.20})$$

$$\tau_{I=4} = \frac{P_{(A)}^2}{2} + \frac{F_{ij} F^{ij}}{4} - i\tilde{\psi}^L \gamma_{(b)} D_{(b)} \Psi^L - J_5^0 \sigma + \partial_k J^k, \quad (\text{C.21})$$

$$\tau_{I=6} = m\tilde{\psi}^L \Psi^L, \quad (\text{C.22})$$

$$\tau_{I=8} = \frac{\varphi^2}{6} R^{(3)}(\mathbf{e}) + \frac{M^2 A_{(b)}^2}{2}, \quad (\text{C.23})$$

here  $\tilde{p}_{(ab)} = \frac{1}{2}(\mathbf{e}_{(a)}^i \tilde{p}_{(b)i} + \mathbf{e}_{(b)}^i \tilde{p}_{(a)i})$ ,  $\tilde{p}_{(b)i} = p_{(b)i} + \mathbf{e}_{(a)}^i \varepsilon_{(c)(a)(b)} J_{(c)}$ ,

$$\mathcal{C} = A_0[\partial_i P_{(A)}^i + eJ_0 - \pi_0] + N_{(b)} T_{(b)t}^0 + \lambda_0 \overline{p_\psi} + \lambda_{(a)} \partial_k \mathbf{e}_{(a)}^k \quad (\text{C.24})$$

denotes the sum of the constraints, where  $J_0 = \tilde{\psi}^L \gamma_0 \Psi^L$  is the zeroth component of the current;  $A_0, N_d, N^i, \lambda_0, \lambda_{(a)}$  are the Lagrange multipliers including the Dirac condition of the minimal 3-dimensional

hyper-surface [13]

$$p_{\tilde{\psi}} = \bar{v} = 0 \rightarrow (\partial_0 - N^l \partial_l) \log \tilde{\psi} = \frac{1}{6} \partial_l N^l, \quad (\text{C.25})$$

that gives a positive value of the Hamiltonian density (C.20), and

$$\begin{aligned} T_{(a)t}^0 &= -\overline{p_\psi} \partial_{(a)} \tilde{\psi} + \frac{1}{6} \partial_{(a)} (\overline{p_\psi} \tilde{\psi}) + 2p_{(b)(c)} \gamma_{(b)|(a)(c)} - \partial_{(b)} p_{(b)(a)} \\ &\quad - \frac{1}{i} \bar{\Psi}^I \gamma^0 \partial_{(a)} \Psi^I - \frac{1}{2i} \partial_{(a)} (\bar{\Psi}^I \gamma^0 \Psi^I) - P_{(A)}^i F_{ik} \mathbf{e}_{(a)}^k - \pi_0 A_{(a)} \end{aligned}$$

are the components of the energy-momentum tensor  $T_{(a)}^0 = T_i^0 \mathbf{e}_{(a)}^i$ .

## D Vacuum creation of particles

### D.1 Particle in Quantum Field Theory

In quantum field theory, the concept of a particle can be associated only with those field variables that are characterized by a positive probability and a positive energy. Negative energies are removed by causal quantization, according to which the creation operator at a negative energy is replaced by the annihilation operator at the respective positive energy. All of the variables that are characterized by a negative probability can be removed according to the scheme of fundamental operator quantization [32]. The results obtained by applying the operator-quantization procedure to massive vector fields in the case of the conformal flat metric are given in [31, 63]<sup>4</sup>.

In order to determine the evolution law for all fields  $\mathbf{v}$ , it is convenient to use the Hamiltonian form of the action functional for their Fourier components  $\mathbf{v}_k^I = \int d^3x e^{i\mathbf{k} \cdot \mathbf{x}} \mathbf{v}^I(\mathbf{x})$ ; that is,

$$\begin{aligned} S &= \int_{x_1^0}^{x_2^0} dx^0 \left\{ \sum_k \left[ \mathbf{p}_k^\perp \partial_0 \mathbf{v}_k^\perp + \mathbf{p}_k^\parallel \partial_0 \mathbf{v}_k^\parallel \right] - P_a \partial_0 a \right. \\ &\quad \left. + N_0 \left[ \frac{P_a^2}{4V_0 \varphi_0^2} - V_0 \rho_{\text{tot}} \right] \right\}, \end{aligned} \quad (\text{D.1})$$

where  $\mathbf{p}_k^\perp, \mathbf{p}_k^\parallel$  are the canonical momenta for, respectively, the transverse and the longitudinal component of vector bosons and  $\rho_{\text{tot}}$  is the sum of the conformal densities of the scalar field obeying the rigid

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<sup>4</sup> The vacuum creation of particles in the conformal flat metric was considered in [92, 93] and, in the time reparametrization-invariant models, in [94].

equation of state and the vector field,

$$\rho_{\text{tot}}(a) = \frac{\varphi_0^2 H_0^2}{a^2} + \rho_v(a), \quad (\text{D.2})$$

$$\rho_v(a) = V_0^{-1}(H^\perp + H^\parallel), \quad (\text{D.3})$$

$H^\perp$  and  $H^\parallel$  being the Hamiltonians for a free field,<sup>5</sup>

$$H^\perp = \sum_k \frac{1}{2} [\mathbf{p}_k^{\perp 2} + \omega^2 v_k^{\perp 2}] , \quad (\text{D.4})$$

$$H^\parallel = \sum_k \frac{1}{2} \left[ \left( \frac{\omega(a, k)}{M_v a} \right)^2 \mathbf{p}_k^{\parallel 2} + (M_v a)^2 v_k^{\parallel 2} \right] .$$

Here, the dispersion relation has the form  $\omega = \sqrt{\mathbf{k}^2 + (M_v a)^2}$ ; for the sake of brevity, we have also introduced the notation  $\mathbf{p}_k^{\parallel 2} \equiv \mathbf{p}_k^\parallel \cdot \mathbf{p}_{-k}^\parallel$ .

Within the reparametrization-invariant models specified by action functionals of the type in (D.1) with the Hamiltonians in (D.4), the concepts of an observable particle and of cosmological particle creation were defined in [94, 95]. We will illustrate these definitions by considering the example of an oscillator with a variable energy. Specifically, we take its Lagrangian in the form

$$\mathcal{L} = p_v \partial_0 v - N_0 \frac{1}{2} [p_v^2 + \omega^2 v^2 - \omega] + \rho_0 (N_0 - 1) . \quad (\text{D.5})$$

The quantity  $H_v = [p_v^2 + \omega^2 v^2]/2$  has the meaning of a “conformal Hamiltonian” as a generator of the evolution of the fields  $v$  and  $p_v$  with respect to the conformal-time interval  $d\eta = N_0 dx^0$ , where the shift function  $N_0$  plays the role of a Lagrange multiplier. The equation for  $N_0$  introduces the density  $\rho_0 = H_v - \omega/2$  in accordance with its definition adopted in the general theory of relativity. In quantum field theory [92, 94, 95], the diagonalization of precisely the conformal Hamiltonian

$$H_v = \frac{1}{2} [p_v^2 + \omega^2 v^2] = \omega \left[ \hat{N}_{\text{part}} + \frac{1}{2} \right] \quad (\text{D.6})$$

specifies both the single-particle energy  $\omega = \sqrt{\mathbf{k}^2 + (M_v a(\eta))^2}$  and the particle-number operator

$$\hat{N}_{\text{part}} = \frac{1}{2\omega} [p_v^2 + \omega^2 v^2] - \frac{1}{2} \quad (\text{D.7})$$

with the aid of the transition to the symmetric variables  $p$  and  $q$  defined as

$$p_v = \sqrt{\omega} p = i \sqrt{\frac{\omega}{2}} (a^+ - a), \quad v = \sqrt{\frac{1}{\omega}} q = \sqrt{\frac{1}{2\omega}} (a^+ + a). \quad (\text{D.8})$$

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<sup>5</sup>In quantum field theory, observables that are constructed from the above field variables form the Poincaré algebra [31, 32]. Therefore, such a formulation, which depends on the reference frame used, does not contradict the general theory of irreducible and unitary transformations of the relativistic group [62, 61].

In terms of the symmetric variables  $p, q$  the particle-number operator takes form

$$\hat{N}_{\text{part}} = \frac{1}{2}[p^2 + q^2] - \frac{1}{2} = a^+ a. \quad (\text{D.9})$$

Upon going over to these variables in the Lagrangian in (D.5), we arrive at

$$\mathcal{L} = p\partial_0 q - pq\partial_0 \Delta^\perp - N_0\omega[\hat{N}_{\text{part}} + 1/2], \quad (\text{D.10})$$

where  $\partial_0 \Delta^\perp = \partial_0 \omega / 2\omega$  and where there appears sources of cosmic particle creation in the form  $pq = i[(a^+)^2 - a^2]/2$ . Here, we give a derivation of these sources for transverse fields, whereas, for longitudinal fields [see Eq.(D.4)], the analogous diagonalization of the Hamiltonian leads to the factor  $\partial_0 \Delta^\parallel = \partial_0 \varphi / \varphi - \partial_0 \omega / 2\omega$ .

In order to diagonalize the equations of motion in terms of the mentioned new variables, it is necessary to apply, to the phase space, the rotation transformation

$$p = p_\theta \cos \theta + q_\theta \sin \theta, \quad q = q_\theta \cos \theta - p_\theta \sin \theta \quad (\text{D.11})$$

and the squeezing phase space transformation

$$p_\theta = \pi e^{-r}, \quad q_\theta = \xi e^{+r}. \quad (\text{D.12})$$

As a result, the Lagrangian in (D.10) assumes the form

$$\begin{aligned} \mathcal{L} = & \pi\partial_0 \xi + \pi\xi[\partial_0 r - \partial_0 \Delta \cos 2\theta] + \\ & + \frac{\pi^2}{2}e^{-2r}[\partial_0 \theta - N_0\omega - \partial_0 \Delta \sin 2\theta] + \frac{\xi^2}{2}e^{2r}[\partial_0 \theta - N_0\omega + \partial_0 \Delta \sin 2\theta]. \end{aligned} \quad (\text{D.13})$$

The equations of motion that are obtained from this Lagrangian,

$$\xi' + \xi[r' - \Delta' \cos 2\theta] + \pi e^{-2r}[\partial_0 \theta - N_0\omega - \partial_0 \Delta \sin 2\theta] = 0, \quad (\text{D.14})$$

$$\pi' - \pi[r' - \Delta' \cos 2\theta] - \xi 2e^{2r}[\partial_0 \theta - N_0\omega + \partial_0 \Delta \sin 2\theta] = 0, \quad (\text{D.15})$$

take a diagonal form,

$$\xi' + \omega_b \pi = 0, \quad -\pi' + \omega_b \xi = 0, \quad (\text{D.16})$$

if

$$\omega_b = e^{-2r}[\omega - \theta' - \Delta' \sin 2\theta] = \frac{\omega - \theta'}{\text{ch} 2r} \quad (\text{D.17})$$

and the rotation parameter  $\theta$  and the squeezing parameter  $r$  satisfy the equations<sup>6</sup>

$$[\theta' - \omega] \text{sh} 2r = -\Delta' \sin 2\theta \text{ch} 2r, \quad r' = \Delta' \cos 2\theta. \quad (\text{D.18})$$

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<sup>6</sup>These equations for transverse and longitudinal bosons coincide completely with the equations for the coefficients of the Bogolyubov transformation  $b = \alpha a + \beta a^+$ ,  $\alpha' - i\omega\alpha = \Delta'\beta$ , derived by using the Wentzel-Kramers-Brillouin method in [93], see Eqs. (9.68) and (9.69) in [93] on page 185 in the Russian edition of this monograph, where it is necessary to make the change of variables specified by the equations  $\Delta' = \frac{\omega^{(1)}}{2}$ ,  $\alpha^* = \exp[i\theta - i \int d\eta\omega] \text{chr}$ ,  $\beta = \exp[-i\theta + i \int d\eta\omega] \text{shr}$ .



By solving these equations, we can find the time dependence of the number of particles produced in cosmic evolution (D.9)

$$\hat{N}_{\text{part}} = \frac{\text{ch}2r - 1}{2} + \text{ch}2r\hat{N}_{\text{q-part}} + \text{sh}2r\frac{\pi^2 - \xi^2}{2}, \quad (\text{D.19})$$

where  $\hat{N}_{\text{q-part}} = [\pi^2 + \xi^2 - 1]/2 = b^+b$  is the number of quasiparticles defined as variables that diagonalize the equation of motion. Since the equation of motion is diagonal, the number of quasiparticles is an integral of the motion, that is, a quantum number that characterizes the quantum state of the Universe. One of these states is the physical vacuum state  $|0\rangle_{\text{sq}}$  of quasiparticles (that is, the squeezed vacuum, which is labelled with the subscript “sq” in order to distinguish it from the vacuum of ordinary particles),

$$b_{\varsigma}|0\rangle_{\text{sq}} = 0 \quad (b = \frac{1}{\sqrt{2}}[\xi + i\pi]). \quad (\text{D.20})$$

In the squeezed-vacuum state, the number of quasiparticles is equal to zeroth

$$_{\text{sq}}\langle 0|\hat{N}_{\text{q-part}}|0\rangle_{\text{sq}} = 0. \quad (\text{D.21})$$

In this case, the expectation value of the particle-number operator (D.19) in the squeezed-vacuum state is

$$_{\text{sq}}\langle 0|\hat{N}_{\text{part}}|0\rangle_{\text{sq}} = \frac{\text{ch}(2r(\eta)) - 1}{2} = \text{sh}^2r(\eta). \quad (\text{D.22})$$

The time dependence of this quantity is found by solving the Bogolyubov equation (D.18). The origin of the Universe is defined as the conformal-time instant  $\eta = 0$ , at which the number of particles and the number of quasiparticles are both equal to zeroth. The resulting set of Eqs. (D.18) becomes closed upon specifying the equation of state and initial data for the number of particles. In just the same way, the number of particles characterized by an arbitrary set of quantum numbers  $\varsigma$ ,

$$\mathcal{N}_{\varsigma}(\eta) = _{\text{sq}}\langle 0|\hat{N}_{\varsigma}|0\rangle_{\text{sq}} = \text{sh}^2r_{\varsigma}(\eta),$$

and produced from the “squeezed” vacuum by the time instant  $\eta$  can be determined by solving an equation of the type in (D.18).

Thus, just the conformal quantities of the theory, such as the energy  $\omega_k = \sqrt{k^2 + M_v^2 a^2}$ , the number particles  $\hat{N}_{\text{part}}$ , the conformal density

$$\rho_v = \sum_k _{\text{sq}}\langle 0|\hat{N}_k|0\rangle_{\text{sq}}\omega_k/V_{(r)}$$

that are associated with observables, in just the same way as the conformal time in observational cosmology is associated with the observed time [64].

## D.2 Physical implications

### D.2.1 Calculation of the Distribution Function

Let us consider the example where the above set of equations is solved for the evolution law in the case of the rigid equation of state,

$$a(\eta) = a_I \sqrt{1 + 2H_I \eta} \quad (a_I^2 H_I = H_0),$$

where  $a_I = a(0)$  and  $H_I$  are initial data at the matter-production instant.

We introduce the dimensionless variables of time  $\tau$  and momentum  $x$  and the coefficient  $\gamma_v$  according to the formulas

$$\tau = 2\eta H_I = \eta/\eta_I, \quad x = \frac{q}{M_I}, \quad \gamma_v = \frac{M_I}{H_I}, \quad (\text{D.23})$$

where  $M_I = M_v(\eta = 0)$  are initial data for the mass. Now the single-particle energy has the form  $\omega_v = H_I \gamma_v \sqrt{1 + \tau + x^2}$ .

The Bogolyubov equations (D.18) can be represented as

$$\tanh(2r_v^{\parallel}) = -\frac{\frac{1}{2(1+\tau)} - \frac{1}{4[(1+\tau) + x^2]}}{\frac{\gamma_v}{2}\sqrt{(1+\tau) + x^2} - \frac{d\theta_v^{\parallel}}{d\tau}} \sin(2\theta_v^{\parallel}), \quad (\text{D.24})$$

$$\frac{dr_v^{\parallel}}{d\tau} = \left[ \frac{1}{2(1+\tau)} - \frac{1}{4[(1+\tau) + x^2]} \right] \cos(2\theta_v^{\parallel}), \quad (\text{D.25})$$

$$\tanh(2r_v^{\perp}) = -\frac{\frac{1}{4[(1+\tau) + x^2]}}{\frac{\gamma_v}{2}\sqrt{(1+\tau) + x^2} - \frac{d}{d\tau}\theta_v^{\perp}} \sin(2\theta_v^{\perp}), \quad (\text{D.26})$$

$$\frac{dr_v^{\perp}}{d\tau} = \frac{1}{4[(1+\tau) + x^2]} \cos(2\theta_v^{\perp}). \quad (\text{D.27})$$

We solved these equations numerically at positive values of the momentum  $x = q/M_I$ , considering that, for  $\tau \rightarrow +0$ , the asymptotic behavior of the solutions is given by  $r(\tau) \rightarrow \text{const} \cdot \tau$  and  $\theta(\tau) = O(\tau)$ . The distributions of longitudinal  $\mathcal{N}^{\parallel}(x, \tau)$  and transverse  $\mathcal{N}^{\perp}(x, \tau)$  vector bosons are given in the Figure 1. for the initial data  $H_I = M_I$  ( $\gamma_v = 1$ ).

From the Figure 1, it can be seen that, for  $x > 1$ , the longitudinal component of the boson distribution is everywhere much greater than than the transverse component, this demonstrating a more copious cosmological creation of longitudinal bosons in relation to transverse bosons. A slow decrease in the longitudinal component as a function of momentum leads to a divergence of the integral for the density of

product particles [63]:

$$n_v(\eta) = \frac{1}{2\pi^2} \int_0^\infty dq q^2 \left[ \mathcal{N}^{\parallel}(q, \eta) + 2\mathcal{N}^{\perp}(q, \eta) \right] \rightarrow \infty. \quad (\text{D.28})$$

### D.2.2 Thermalization of Bosons

The divergence of the integral (D.28) stems from idealizing the problem of the production of a pair of particles in a finite volume for a system where there are simultaneous interactions associated with the removal of fields having a negative probability and where identical particles affect one another (so-called exchange effects). In this case, it is well known [96], that one deals with the production not a pair but a set of Bose – particles, which acquires, owing to the aforementioned interactions, the properties of a statistical system. As a model of such a statistical system, we consider here a degenerate Bose-Einstein gas, whose distribution function has the form (we use the system of units where the Boltzmann constant is  $k_B = 1$ )

$$\mathcal{F}(T_v, q, M_v(\eta), \eta) = \left\{ \exp \left[ \frac{\omega_v(\eta) - M_v(\eta)}{T_v} \right] - 1 \right\}^{-1}, \quad (\text{D.29})$$

where  $T_v$  is the boson temperature. We set apart the problem of theoretically validating such a statistical system and its thermodynamic exchange, only assuming fulfillment of specific conditions ensuring its existence. In particular, we can introduce the notion of the temperature  $T_v$  only in an equilibrium system. A thermal equilibrium is thought to be stable if the time within which the vector-boson temperature  $T_v$  is established, that is, the relaxation time [97]

$$\eta_{\text{rel}} = [n(T_v)\sigma_{\text{scat}}]^{-1} \quad (\text{D.30})$$

(expressed in terms of their density  $n(T_v)$  and the scattering cross section  $\sigma_{\text{scat.}} \sim 1/M_I^2$ ), does not exceed the time of vector-boson-density formation owing to cosmological creation, the latter time being controlled by the primordial Hubble parameter  $\eta_v = 1/H_I$ . From formula (D.30) it follows, that the particle-number density is proportional to the product of the Hubble parameter and the mass squared, that is an integral of the motion in the present example:

$$n(T_v) = n(T_v, \eta_v) \simeq C_H H_I M_I^2, \quad (\text{D.31})$$

where  $C_H$  is a constant. The expression for the density  $n(T_v, \eta)$  in Eq. (D.31) assumes the form

$$n_v(T_v, \eta) = \frac{1}{2\pi^2} \int_0^\infty dq q^2 \mathcal{F}(T_v, q, M(\eta), \eta) \left[ \mathcal{N}^{\parallel}(q, \eta) + 2\mathcal{N}^{\perp}(q, \eta) \right]. \quad (\text{D.32})$$

Here, the probability of the production of a longitudinal and a transverse boson with a specific momentum in an ensemble featuring exchange interaction is given (in accordance with the multiplication law for probabilities) by the product of two probabilities, the probability of their cosmological creation,  $\mathcal{N}^{||,\perp}$  and the probability of a single-particle state of vector bosons obeying the Bose-Einstein distribution (D.29).

A dominant contribution to the integral (D.32) from the region of high momenta (in the above idealized analysis disregarding the Boltzmann factor, this resulted in a divergence) implies the relativistic temperature dependence of the density,

$$n(T_v, \eta_v) = C_T T_v^3, \quad (\text{D.33})$$

where  $C_T$  is a coefficient. A numerical calculation of the integral (D.32) for the values  $T_v = M_I = H_I$ , which follow from the assumption about the choice of initial data ( $C_T = C_H$ ), reveals that this integral (D.32) is weakly dependent on time in the region  $\eta \geq \eta_v = H_I^{-1}$  and, for the constant  $C_T$ , yields the value

$$C_T = \frac{n_v}{T_v^3} = \frac{1}{2\pi^2} \left\{ [1, 877]^{||} + 2[0, 277]^{\perp} = 2, 431 \right\}, \quad (\text{D.34})$$

where the contributions of longitudinal and transverse bosons are labeled with the superscripts ( $||$ ,  $\perp$ ), respectively.

On the other hand, the lifetime  $\eta_L$  of product bosons in the early Universe in dimensionless units  $\tau_L = \eta_L/\eta_I$ , where  $\eta_I = (2H_I)^{-1}$ , can be estimated by using the equation of state  $a^2(\eta) = a_I^2(1 + \tau_L)$  and the  $W$ -boson lifetime within the Standard Model. Specifically, we have

$$1 + \tau_L = \frac{2H_I \sin^2 \theta_{(W)}}{\alpha_{\text{QED}} M_W(\eta_L)} = \frac{2 \sin^2 \theta_{(W)}}{\alpha_{\text{QED}} \gamma_v \sqrt{1 + \tau_L}}, \quad (\text{D.35})$$

where  $\theta_{(W)}$  is the Weinberg angle,  $\alpha_{\text{QED}} = 1/137$  is the fine-structure constant, and  $\gamma_v = M_I/H_I \geq 1$ .

From the solution to Eq. (D.35),

$$\tau_L + 1 = \left( \frac{2 \sin^2 \theta_{(W)}}{\gamma_v \alpha_{\text{QED}}} \right)^{2/3} \simeq \frac{16}{\gamma_v^{2/3}} \quad (\text{D.36})$$

it follows that, at  $\gamma_v = 1$ , the lifetime of product bosons is an order of magnitude longer than the Universe relaxation time:

$$\tau_L = \frac{\eta_L}{\eta_I} \simeq \frac{16}{\gamma_v^{2/3}} - 1 = 15. \quad (\text{D.37})$$

Therefore, we can introduce the notion of the vector-boson temperature  $T_v$ , which is inherited by the final vector boson decay products (photons). According to currently prevalent concepts, these

photons form cosmic microwave background radiation in the Universe. Indeed, suppose that one photon comes from the annihilation of the products of  $W^\pm$ -boson decay and that the other comes from  $Z$ -bosons. In view of the fact that the volume of the Universe is constant within the evolution model being considered, it is then natural to expect that the photon density coincides with the boson density [63]

$$n_\gamma = T_\gamma^3 \frac{1}{\pi^2} \{2.404\} \simeq n_v. \quad (\text{D.38})$$

On the basis of (D.31), (D.33), (D.34) and (D.38) we can estimate the temperature  $T_\gamma$  of cosmic microwave background radiation arising upon the annihilation and decay of  $W^+$  and  $Z$ -bosons:

$$T_\gamma \simeq \left[ \frac{2.431}{2.404 \cdot 2} \right]^{1/3} T_v = 0.8 T_v, \quad (\text{D.39})$$

taking into account that the temperature of vector-bosons  $T_v = [H_I M_I^2]^{1/3}$  is an invariant quantity in the described model. This invariant can be estimated at

$$T_v = [H_I M_I^2]^{1/3} = [H_0 M_W^2]^{1/3} = 2.73/0.8 K = 3.41 K \quad (\text{D.40})$$

which is a value that is astonishingly close to the observed temperature of cosmic microwave background radiation. In the present case, this directly follows, as is seen from the above analysis of our numerical calculations, from the dominance of longitudinal vector bosons with high momenta and from the fact that the relaxation time is equal to the inverse Hubble parameter. The inclusion of physical processes, like the heating of photons owing to electron-positron annihilation  $e^+ e^-$  [98] amounts to multiplying the photon temperature (D.39) by  $(11/4)^{1/3} = 1.4$  therefore, we have

$$T_\gamma(e^+ e^-) \simeq (11/4)^{1/3} 0.8 T_v = 2.77 K. \quad (\text{D.41})$$

We note that, in other models [99], the fluctuations of the product-particle density are related to primary fluctuations of cosmic microwave background radiation [100].

### D.2.3 Inverse Effect of Product Particles on the Evolution of the Universe

The equation of motion  $\varphi'^2(\eta) = \rho_{\text{tot}}(\eta)$ , with the Hubble parameter defined as  $H = \varphi'/\varphi$ , means that, at any instant of time, the energy density in the Universe is equal to the so-called critical density; that is

$$\rho_{\text{tot}}(\eta) = H^2(\eta) \varphi^2(\eta) \equiv \rho_{\text{cr}}(\eta).$$

The dominance of matter obeying the extremely rigid equation of state implies the existence of an approximate integral of the motion in the form

$$H(\eta)\varphi^2(\eta) = H_0\varphi_0^2.$$

On this basis, we can immediately find the ratio of the product-vector-boson energy,  $\rho_v(\eta_I) \sim T^4 \sim H_I^4 \sim M_I^4$ , to the density of the Universe in the extremely rigid state,  $\rho_{\text{tot}}(\eta_I) = H_I^2\varphi_I^2$ ,

$$\frac{\rho_v(\eta_I)}{\rho_{\text{tot}}(\eta_I)} = \frac{M_I^2}{\varphi_I^2} = \frac{M_W^2}{\varphi_0^2} = y_v^2 = 10^{-34}. \quad (\text{D.42})$$

This value indicates that the inverse effect of product particles on the evolution of the Universe is negligible.

The primordial mesons before their decays polarize the Dirac fermion vacuum and give the baryon asymmetry frozen by the CP – violation, so that  $n_b/n_\gamma \sim X_{CP} \sim 10^{-9}$  and  $\Omega_b \sim \alpha_{\text{qed}}/\sin^2\theta_{(W)} \sim 0.03$ .

#### D.2.4 Baryon-antibaryon Asymmetry of Matter in the Universe

In each of the three generations of leptons (e,  $\mu$ ,  $\tau$ ) and color quarks, we have four fermion doublets-in all, there are  $n_L = 12$  of them. Each of 12 fermion doublets interacts with the triplet of non-Abelian fields  $A^1 = (W^{(-)} + W^{(+)})/\sqrt{2}$ ,  $A^2 = i(W^{(-)} - W^{(+)})/\sqrt{2}$ , and  $A^3 = Z/\cos\theta_{(W)}$ , the corresponding coupling constant being  $g = e/\sin\theta_{(W)}$ .

It is well known that, because of a triangle anomaly, W- and Z- boson interaction with lefthanded fermion doublets  $\psi_L^{(i)}$ ,  $i = 1, 2, \dots, n_L$ , leads to a nonconservation of the number of fermions of each type ( $i$ ) [101, 102, 103],

$$\partial_\mu j_{L\mu}^{(i)} = \frac{1}{32\pi^2} \text{Tr} \hat{F}_{\mu\nu} * \hat{F}_{\mu\nu}, \quad (\text{D.43})$$

where  $\hat{F}_{\mu\nu} = -iF_{\mu\nu}^a g_W \tau_a/2$  is the strength of the vector fields,  $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc}A_\mu^b A_\nu^c$ .

Taking the integral of the equality in (D.43) with respect to the four-dimensional variable  $x$ , we can find a relation between the change  $\Delta F^{(i)} = \int d^4x \partial_\mu j_{L\mu}^{(i)}$  the fermion number  $F^{(i)} = \int d^3x j_0^{(i)}$  and the Chern-Simons functional,  $N_{CS} = \frac{1}{32\pi^2} \int d^4x \text{Tr} \hat{F}_{\mu\nu} * \hat{F}_{\mu\nu}$ :

$$\Delta F^{(i)} = N_{CS} \neq 0, \quad i = 1, 2, \dots, n_L. \quad (\text{D.44})$$

The equality in (D.44) is considered as a selection rule – that is, the fermion number changes identically for all fermion types:  $N_{CS} =$

$\Delta L^e = \Delta L^\mu = \Delta L^\tau = \Delta B/3$ ; at the same time, the change in the baryon charge  $B$  and the change in the lepton charge  $L = L^e + L^\mu + L^\tau$  are related to each other in such a way that  $B - L$  is conserved, while  $B + L$  is not invariant. Upon taking the sum of the equalities in (D.44) over all doublets, one can obtain  $\Delta(B + L) = 12N_{CS}$  [103].

We can evaluate the expectation value of the Chern-Simons functional (D.44) (in the lowest order of perturbation theory in the coupling constant) in the Bogolyubov vacuum  $b|0\rangle_{\text{sq}} = 0$ . Specifically, we have

$$N_{CS} = N_W + N_Z \equiv - \sum_{v=W,Z} \int_0^{\eta_{L_v}} d\eta \int \frac{d^3x}{32\pi^2} {}_{\text{sq}}\langle 0 | \text{Tr} \hat{F}_{\mu\nu}^v {}^* \hat{F}_{\mu\nu}^v | 0 \rangle_{\text{sq}}, \quad (\text{D.45})$$

where  $\eta_{L_W}$  and  $\eta_{L_Z}$  are the W- and the Z-boson lifetime, and  $N_W$  and  $N_Z$  are the contributions of primordial W and Z bosons, respectively. The integral over the conformal spacetime bounded by three-dimensional hypersurfaces  $\eta = 0$  and  $\eta = \eta_L$  is given by

$$N_v = \beta_v \frac{V_0}{2} \int_0^{\eta_{L_v}} d\eta \int_0^\infty dk |k|^3 R_v(k, \eta)$$

where  $v = W, Z$ ;

$$\beta_W = \frac{4\alpha_{\text{QED}}}{\sin^2 \theta_{(W)}}, \quad \beta_Z = \frac{\alpha_{\text{QED}}}{\sin^2 \theta_{(W)} \cos^2 \theta_{(W)}}, \quad (\text{D.46})$$

and the rotation parameter

$$R_v = -\sinh(2r) \sin(2\theta)$$

is specified by relevant solutions to the Bogolyubov equations (D.26). Upon a numerical calculation of this integral, we can estimate the expectation value of the Chern-Simons functional in the state of primordial bosons.

At the vector-boson-lifetime values of  $\tau_{L_W} = 15$ ,  $\tau_{L_Z} = 30$ , this yields the following result at  $n_\gamma \simeq n_v$

$$\frac{N_{CS}}{V_{(r)}} = \frac{(N_W + N_Z)}{V_{(r)}} = \frac{\alpha_{\text{QED}}}{\sin^2 \theta_{(W)}} T^3 \left( 4 \times 1.44 + \frac{2.41}{\cos^2 \theta_{(W)}} \right) = 1.2 n_\gamma. \quad (\text{D.47})$$

On this basis, the violation of the fermion-number density in the cosmological model being considered can be estimated as [63, 64]

$$\frac{\Delta F^{(i)}}{V_{(r)}} = \frac{N_{CS}}{V_{(r)}} = 1.2 n_\gamma, \quad (\text{D.48})$$

where  $n_\gamma = 2,402 \times T^3/\pi^2$  is the number density of photons forming cosmic microwave background radiation.

According to Sakharov [104] this violation of the fermion number is frozen by CP nonconservation, this leading to the baryon-number density

$$n_b = X_{\text{CP}} \frac{\Delta F^{(i)}}{V_{(r)}} \simeq X_{\text{CP}} n_\gamma . \quad (\text{D.49})$$

where the factor  $X_{\text{CP}}$  is determined by the superweak interaction of  $d$  and  $s$  quarks, which is responsible for CP violation experimentally observed in  $K$ -meson decays [105].

From the ratio of the number of baryons to the number of photons, one can deduce an estimate of the superweak-interaction coupling constant:  $X_{\text{CP}} \sim 10^{-9}$ . Thus, the evolution of the Universe, primary vector bosons, and the aforementioned superweak interaction [105] (it is responsible for CP violation and is characterized by a coupling-constant value of  $X_{\text{CP}} \sim 10^{-9}$ ) lead to baryon-antibaryon asymmetry of the Universe, the respective baryon density being

$$\rho_b(\eta = \eta_L) \simeq 10^{-9} \times 10^{-34} \rho_{\text{cr}}(\eta = \eta_L). \quad (\text{D.50})$$

In order to assess the further evolution of the baryon density, one can take here the W-boson lifetime for  $\eta_L$ .

Upon the decay of the vector bosons in question, their temperature is inherited by cosmic microwave background radiation. The subsequent evolution of matter in a stationary cold universe is an exact replica of the well-known scenario of a hot universe [106], since this evolution is governed by conformally invariant mass-to-temperature ratios  $m/T$ .

Formulas (D.35), (2.39), and (D.50) make it possible to assess the ratio of the present-day values of the baryon density and the density of the scalar field, which plays the role of primordial conformal quintessence in the model being considered. We have

$$\Omega_b(\eta_0) = \frac{\rho_b(\eta_0)}{\rho_{\text{cr}}(\eta_0)} = \left[ \frac{\varphi_0}{\varphi_L} \right]^3 = \left[ \frac{\varphi_0}{\varphi_I} \right]^3 \left[ \frac{\varphi_I}{\varphi_L} \right]^3, \quad (\text{D.51})$$

where we have considered that the baryon density increases in proportion to the mass and that the density of the primordial quintessence decreases in inverse proportion to the mass squared. We recall that the ratio  $[\varphi_0/\varphi_I]^3$  is approximately equal to  $10^{43}$  and that the ratio  $[\varphi_I/\varphi_L]^3$  is determined by the boson lifetime in (D.36) and by the equation of state  $\varphi(\eta) \sim \sqrt{\eta}$ . On this basis, we can estimate  $\Omega_b(\eta_0)$  at

$$\Omega_b(\eta_0) = \left[ \frac{\varphi_0}{\varphi_L} \right]^3 10^{-43} \sim 10^{43} \left[ \frac{\eta_I}{\eta_L} \right]^{3/2} 10^{-43} \sim \left[ \frac{\alpha_{\text{QED}}}{\sin^2 \theta_{(W)}} \right] \sim 0.03, \quad (\text{D.52})$$



which is compatible with observational data [107].

Thus, the general theory of relativity and the Standard Model, which are supplemented with a free scalar field in a specific reference frame with the initial data  $\varphi_I = 10^4$   $H_I = 2.7$  K, do not contradict the following scenario of the evolution of the Universe within conformal cosmology [63, 64]:

$\eta \sim 10^{-12}s$ , creation of vector bosons from a “vacuum”;

$10^{-12}s < \eta < 10^{-11} \div 10^{-10}s$ , formation of baryon-antibaryon asymmetry;

$\eta \sim 10^{-10}s$ , decay of vector bosons;

$10^{-10}c < \eta < 10^{11}s$ , primordial chemical evolution of matter;

$\eta \sim 10^{11}s$ , recombination or separation of cosmic microwave background radiation;

$\eta \sim 10^{15}s$ , formation of galaxies;

$\eta > 10^{17}s$ , terrestrial experiments and evolution of supernovae.

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